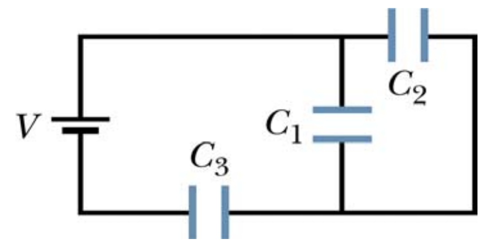
Phyx 135-2, Midterm I (1 pm), Fall 2015 **Name: ______________________________**

1) In the circuit at right, $C_1 = 4\ \mu F$, $C_2 = 6\ \mu F$, $C_3 = 8\ \mu F$, and $V = 9$ volts.



a) (5 points) What is the total capacitance of the circuit?
b) (5 points) What is the voltage across C_3?

Solutions

a) C_1 and C_2 are in parallel with each other. Their capacitances therefore add, so $C_{12} = 10\ \mu F$.

C_{12} and C_3 are in series, so their capacitances must be added using the "one over" rule:
$1 / C_{eq} = (1 / 10\ \mu F) + (1 / 8\ \mu F) = 225{,}000$ so $C_{eq} = 4.444\ \mu F$.

b) The charge on the entire circuit is given by $q = C_{eq}V$, so $q = (4.44\ \mu F)(9 \text{ volts}) = 40\ \mu C$. Since C_3 is in series, it must have the same charge on it. Thus the voltage across it will be $V = q/C_3 = 40\ \mu C / 8\ \mu F = 5$ volts.

2) (8 points) Suppose three charged particles form a triangle:

Particle 1 with q_1 = 80 nC is at xy coordinates (0, 3 mm);

Particle 2 with q_2 = 80 nC is at xy coordinates (0, –3 mm); and

Particle 3 with q_3 = 18 nC is at xy coordinates (4, 0 mm).

In unit-vector notation, what is the electrostatic force on Particle 3 due to the other two particles? (Assume $k = 8.99 \times 10^9$ N m^2 / C^2.)

Solution

From Coulomb's Law, we have $F_{12} = (kq_1q_2 / r^2)\ \boldsymbol{r}$. Drawing a little picture (or just considering the coordinates given above) reveals that the triangle is symmetric around the x-axis. Therefore the net force in the y-direction on Particle 3 is zero.

In the x-direction, both particles 1 and 2 will exert equal force on Particle 3. This will be:
$F = [(8.99 \times 10^9)(80\text{ nC})(18\text{ nC}) / (0.003^2 + 0.004^2)]\ [0.004 / (0.003^2 + 0.004^2)^{1/2}]$, where the second term in [ ] brackets is the unit vector $\boldsymbol{r}$ in the $\boldsymbol{i}$ direction.

Punching in the numbers yields F = 2(0.5178)(0.004/0.005) = 0.8285 $\boldsymbol{i}$ N.

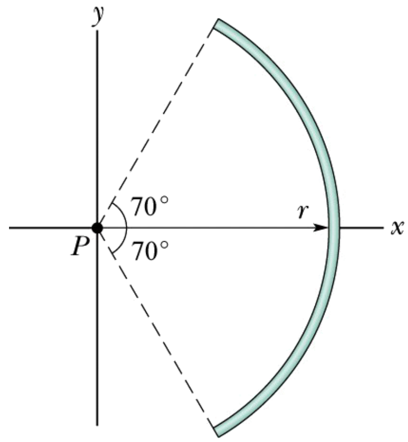
3) (8 points) Suppose you have a region of space where the electric potential is given by: $V = A(x/a)^{4/3}$, where A = 80 volts and a = 3 cm. If you were to place an electron at x = 2 cm, what would be the magnitude of the force operating on it? ($e = 1.6 \times 10^{-19}$ C for the electron)

Solution

The force on the electron will be F = eE. In this case, we need to find the electric field by differentiating the potential. So, $E = A(^4/_3)x^{1/3} / a^{4/3} = (80)(^4/_3)(0.02)^{1/3} / 0.03^{4/3}$ = 3106 volts/m.

We have $F = eE = (1.6 \times 10^{-19})(3106) = 4.97 \times 10^{-16}$ N

4) A glass rod is curved into an arc with radius r = 4 cm. As shown at right, the rod occupies a total angle of 140° if placed symmetrically on the x-axis. It has a charge of q = 5 nC on it.



Main option (14 points) *Derive* the magnitude of the electric field at the center point P, starting with the definition of the electric field for a point charge. You must show and/or explain all steps to receive full credit.

Alternative option (for 6 points) If you have no clue how to derive the main option, then you may simply write down an expression for the electric field for an arc (presumedly from your memory or notes) and calculate the magnitude of the electric field at the center point P from that.

Solution

The full derivation is shown on the handout "Electric Field for an Arc", which has been on the class website since the first day of class, so I will not reproduce it here.

The relevant formula is $E = (kq/r^2)(\sin\theta / \theta)$. In this case q = 70° = 70π/180 rad.

$E = [(8.99 \times 10^9)(5 \times 10^{-9}) / 0.04^2]\ [180 \sin(70°) / 70\pi]$ = 21,600 volts/m