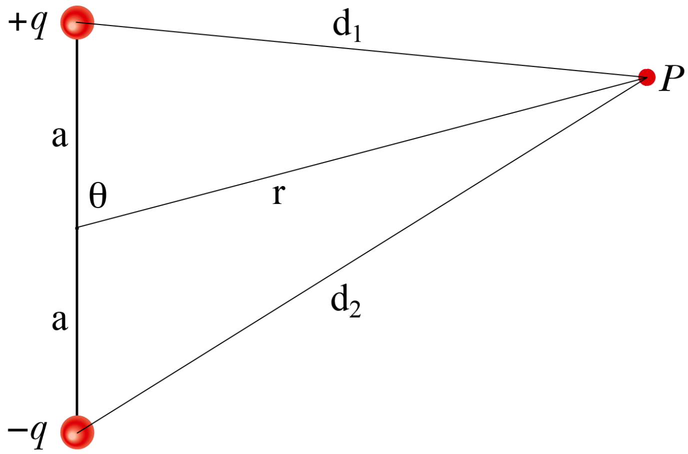

Electric Field of a Dipole

Suppose we have the arrangement shown at left. Two charges of equal magnitude q but opposite sign are located at equal distances a from their midpoint. What is the electric field E at any point P located at a distance r from the midpoint, if a is very *very* small compared to r?

Solution: If a is very small compared to r (and therefore also very small compared to d_1 and d_2), then we can say that d_1 and d_2 point in approximately the same direction to any point P that is far away, never mind how they may look in the figure. Thus the magnitude of the electric field at point P will just be $E = E_1 + E_2 = kq / d_1^2 - kq / d_2^2$. For a non-right triangle, the law of cosines (see page A-9 in your textbook) tells us that $d_1^2 = a^2 + r^2 - 2ar \cos\theta$, and similiarly that $d_2^2 = a^2 + r^2 - 2ar \cos(180^\circ - \theta)$. But since $\cos(180^\circ - \theta) = -\cos\theta$, we can also say $d_2^2 = a^2 + r^2 + 2ar \cos\theta$. This gives us:

$$E = \frac{kq}{a^2 + r^2 - 2ar\cos\theta} - \frac{kq}{a^2 + r^2 + 2ar\cos\theta}.$$

a^2 is very small compared to either r^2 or $2ar \cos\theta$, so we will drop it. If we factor kq / r^2 to the outside, then we are left with:

$$E = \frac{kq}{r^2} \left\{ \frac{1}{1 - 2(a/r)\cos\theta} - \frac{1}{1 + 2(a/r)\cos\theta} \right\}.$$

To make life simple, I will let $X = 2(a/r) \cos\theta$. Then the quantity inside the curly brackets becomes:

$\frac{1}{1 - X} - \frac{1}{1 + X}$, and if we put this over a common denominator we have:

$(1 + X - 1 + X) / (1 - X + X - X^2) = 2X / (1 - X^2)$. But X^2 is very small compared to one, so we can just drop it and we are left with $2X = 4a \cos\theta / r$.

This means $E = 4\,kqa \cos\theta / r^3 = 2kp \cos\theta / r^3$, where I have set $p = 2aq$. As pointed out in the textbook, the quantity $p = 2aq$ (i.e., the distance between the charges multiplied by the charge) is all that you can measure if you are far away from the dipole. In other words, $\frac{1}{2} q \times 2a = qa$, so if my original diagram had shown half the charge but double the distance, we have still have obtained the same answer for E. (Another way of looking at this is to say that we basically assumed a is too small to be measured directly. The only way to measure a is to measure p, and then independently find a value for the charge.) p is called the *dipole moment* of the dipole.