Physics 135-2, Final Exam
Fall 2016 (1 pm Lecture)

Name: ______________________________

Discussion Section: __________________

1) Suppose an electric generator is producing an alternating emf of $V_0 \sin(\omega t)$, where $V_0 = 100$ volts, and $\omega = 2\pi(50 \text{ Hz}) = 314.16$ rad/s. It is connected to a series RLC circuit with $R = 6\ \Omega$, $L = 0.038$ H, and $C = 1.6$ mF.

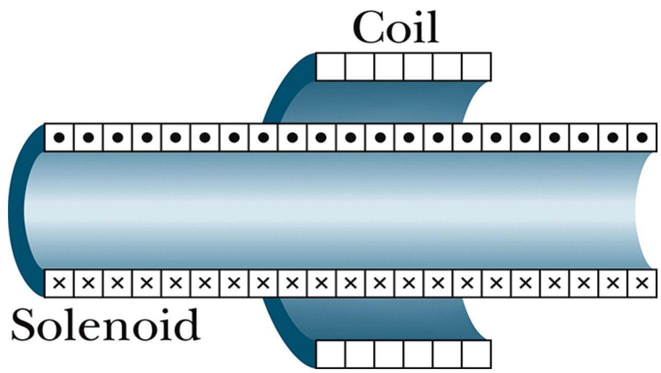
a) (5 points) What is the maximum current that can be flowing in this circuit at any given time?

Solution: The circuit's impedance is given by $Z^2 = R^2 + [(1/\omega C) - \omega L]^2$. Inserting numbers: $Z = [6^2 + (1.989 - 11.938)^2]^{1/2} = 11.6\ \Omega$. The maximum current is 100 / 11.6 = 8.62 amps.

b) (5 points) How long (in seconds) after the voltage from the generator reaches a maximum will the current in the circuit reach a maximum?

Solution: The current will have a phase angle (compared to the emf) of $\tan \phi = (X_L - X_C) / R = (11.938 - 1.989) / 6 = 1.658$, or $\phi = 58.9$ degrees. This means that the current is 58.9 / 360 = 0.164 of a cycle behind the emf. In this case, one cycle takes 1 / 50 Hz = 0.02 s, so the current will peak (0.164)(0.02) = 3.27×10^{-3} s after the emf.

2) (10 points) You have a short coil of wire with a radius of a = 4 cm which has N = 6 turns of wire in it. Inside and coaxial with the coil, you have a very long solenoid with n = 20 turns/cm and a radius of b = 2 cm. A cut-away view is shown at right.



A current of $i(t) = I_0[(3t/\tau) + (t^2/\tau^2)]$ is flowing through the solenoid, with I_0 = 10 amps and τ = 0.10 sec.

What will be the emf in the outer coil at t = 0.50 sec?

Solution

From Faraday's law of induction, we know that $\mathcal{E} = N\, d\Phi/dt$. In this case we have $\Phi = BA\cos\theta$ where the A in question is that of the inner solenoid, because that is the only place where there is a magnetic field. Noting that B for a solenoid is $\mu_o in$, we have:

$\Phi = B(\pi r^2)\cos(0°) = \mu_o\, I_0[(3t/\tau) + (t^2/\tau^2)](20/0.01)\pi(0.02)^2 = (3.16 \times 10^{-5})[(3t/\tau) + (t^2/\tau^2)]$

Then $\mathcal{E} = N\, d\Phi/dt = 6(3.16 \times 10^{-5})[(3/\tau) + (2t/\tau^2)] = 6(3.16 \times 10^{-5})[(3/0.1) + (2(0.5)/0.1^2)] =$ 0.025 volts.

3) (10 points) A negative charge of $Q_1 = -2$ nC is at the origin of an x-y coordinate system. A positive charge of $Q_2 = +4$ nC is setting at x = 1 meter along the x-axis. If an electron (initially at rest) is setting at x = 0.5 m in this system, then is allowed to move freely to x = 0.6 m, how fast will it be moving when it reaches 0.6 m? (electron mass = 9.11×10^{-31} kg)

Solution

The easy way to solve this problem is to use the electric potential. We know that $V = kq/r$ for each charge, so the electric potential of the electron is initially:
$V = k(-2\text{ nC}/0.5\text{ m} + 4\text{ nC}/0.5\text{ m}) = (9 \times 10^9)(2 \times 10^{-9})/0.5 = 36$ volts

After it moves, the electron will have an electric potential of:
$V = k(-2\text{ nC}/0.6\text{ m} + 4\text{ nC}/0.4\text{ m}) = (9 \times 10^9)(10^{-9})(-2/0.6 + 4/0.4) = 60$ volts

Thus (because it has a negative charge), the potential energy of the electron will drop from −36 eV to −60 eV, which means its kinetic energy must rise by 24 eV. Converting units, we have: $E_K = e(V_2 - V_1) = 24\text{ eV} = (1.602 \times 10^{-19})(24) = 3.845 \times 10^{-18}$ J.

Using $E_K = ½ mv^2$ gives us $v = (2E/m)^{1/2} = [(2)(3.845 \times 10^{-18})/(9.11 \times 10^{-31})]^{1/2} =$
2.9×10^6 m/s.

4) (10 points) A hollow cylindrical conductor with inner radius 2.0 mm and outer radius 4.0 mm carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field 3.0 mm from the central axis of the wire?

Solution

We can use Ampere's Law, which tells us that the line integral of B•ds = $\mu_0 I_{enclosed}$. Because we have cylindrical symmetry, we can rewrite the integral of B•ds as $B(2\pi r)$.

That leaves us with I_{enc}. The cross-sectional area of the cylinder is $\pi(b^2 - a^2)$, where b and a are the outer and inner radii, respectively. Therefore the 2D current density σ is (24 amps)/$\pi(b^2 - a^2)$, or $\sigma = 24/\pi(0.004^2 - 0.002^2) = 6.366 \times 10^5$ amp/m^2. The amount of this current which is enclosed within a radius of 3 mm will then be $\sigma\,\pi(0.003^2 - 0.002^2) = (6.366 \times 10^5)\pi(5 \times 10^{-6}) = 10$ amps.

But the inner wire is carrying 24 amps in the opposite direction, so the net current enclosed by our Amperian loop is 24 – 10 = 14 amps. We thus have $B = \mu_0 I_{enc} / 2\pi r = (4\pi \times 10^{-7})(14)/2\pi(0.003) = (2 \times 10^{-7})(14)/(0.003) = 9.33 \times 10^{-4}$ T.

5) Unpolarized light with an intensity of 10 W/m^2 is sent through an ordinary polarizing sheet which is one meter on a side.

5a) (6 points) What is the *maximum* amplitude (not rms) of the electric field component of the transmitted light?

Solution

We have $I = E^2/c\mu_0$, so $E = (c\mu_o I)^{1/2}$, where $I = 5\ W/m^2$ because the polarizing sheet will only transmit one-half of the incident unpolarized light. Also, this formula gives us the rms value of E, whereas the problem asks for the maximum E, so we must multiply by $\sqrt{2}$. We have:

$E = \sqrt{2}[(3 \times 10^8)(4\pi \times 10^{-7})(5)]^{1/2}$ = 61.4 volts

5b) (4 points) What is the total force acting on the sheet due to its absorbing some of the light?

Solution

We have $p = I/c$, so $F = pA = IA/c = 5/(3 \times 10^8)$ = 1.67×10^{-8} N

6) Suppose you have a single electron moving in a circle of radius 5.29×10^{-11} m at a speed of 2.19×10^{6} m/s. This can be thought of as a current loop.

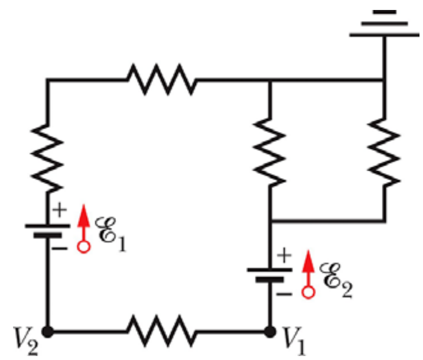
6a) (5 points) Given that one amp is the same as one coulomb of charge passing a given point per second, how many amps is the rotating electron equivalent to?

Solution
One amp is a coulomb per second, and one electron contains 1.6×10^{-19} C, so one amp is equivalent to $1 / 1.6 \times 10^{-19} = 6.25 \times 10^{18}$ electrons per second. The electron will circle the loop $(2.19 \times 10^{6}) / 2\pi(5.29 \times 10^{-11}) = 6.59 \times 10^{15}$ times per second, so it is equivalent to $(6.59 \times 10^{15}) / (6.25 \times 10^{18}) =$ 1.05 mA.

6b) (5 points) If the electron lies in a uniform magnetic field of B = 7.1 mT, what is the maximum possible torque that can be produced on the loop by the field?

Solution
We have $\tau = \mu \times B$, so the maximum possible torque is just $\mu B = NiAB = i(\pi r^2)B$. We have $\tau = (1.05 \times 10^{-3})\pi(5.29 \times 10^{-11})^2(7.1 \times 10^{-3}) =$ 6.58×10^{-26} N-m.

7) (10 points) In the circuit at right, $\mathcal{E}_1 = 5.0$ v, $\mathcal{E}_2 = 12$ v, and the resistors are all 2 Ω. The zero of potential is defined to be at the upper right-hand corner. What are V_1 and V_2?



Solution

The two resistors at the upper right are in parallel, so they will add to give ½ + ½ = 1 Ω of resistance. The rest of the resistors are all in series, so the entire loop has a resistance of 7 Ω. The batteries are working against each other, so we have a net emf of 12 – 5 = 7 volts in the circuit, which means that the current flowing is just 7 / 7 = 1 amp. The current is flowing counter-clockwise because $\mathcal{E}_2$ is greater than $\mathcal{E}_1$.

If we start at the grounding point and move counter-clockwise (to follow the current), then we first cross the top resistor and lose V = IR = (1)(2) = 2 volts. We next go through the left-hand resistor and lose two more volts. Then we move *against* $\mathcal{E}_1$, which means we will lose 5 more volts. This thus yields –2 – 2 – 5 = –9 volts for the voltage at V_2. Crossing the next resistor at the bottom moves us another two volts down, to –11 volts at V_1.

Multiple Choice. Select the one best answer. 2 points each.

_____ **8)** Which of the following lists correctly gives the ordering of electromagnetic radiation from longest wavelength to shortest?

A) AM radio, red light, blue light, gamma-rays
B) microwaves, TV channels, green light, infrared
C) 60-Hz noise, X-rays, ultraviolet, blue light
D) infrared, maritime radio, red light, X-rays
E) infrared, blue light, red light, gamma-rays
F) AM radio, FM radio, gamma-rays, yellow light

Answer: A

_____ **9)** If the maximum magnetic field in an electromagnetic wave is 10^{-4} T, then the maximum electric field in that same wave is:

A) 3×10^{-2} V/m	**B)** 3×10^{-1} V/m	**C)** 3×10^{1} V/m
D) 3×10^{2} V/m	**E)** 3×10^{3} V/m	**F)** 3×10^{4} V/m

Answer: F. $E = cB = (3 \times 10^{8})(10^{-4}) = 3 \times 10^{4}$ V/m.

_____ **10)** The Curie temperature is the temperature at which:

A) Iron melts.
B) All metals become magnets.
C) A ferromagnet becomes a paramagnet.
D) A diamagnetic material stops being magnetic.
E) Magnetic monopoles might theoretically exist.
F) Small magnetic domains fuse into one large domain.

Answer: C. A ferromagnet can no longer form domains above the Curie temperature, but it can still react to a magnetic field, so that means it has become a paramagnet.

_____ **11)** The Hall Effect refers to the fact that:

A) A current-carrying strip in a magnetic field develops a voltage across the strip.
B) An electron in a uniform magnetic field will move in a circle.
C) $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ for all charged particles.
D) An electron moving into a magnetic field tends to be reflected by the field.
E) A current loop in a magnetic field always experiences a torque.
F) The total magnetic flux through any surface must always be zero.

Answer: A

______ **12)** Which of the following statements about the "displacement current" is *false*?

A) It has the units of amps.
B) Its magnitude is $\varepsilon_0\, d\Phi_E/dt$.
C) It indirectly plays a role in the production of light.
D) It is an actual current, but it is always extremely small.
E) It can be created by charging a capacitor.
F) It is a critical part of Maxwell's equations.

Answer: D. The displacement current never involves any actual movement of electric charge.