Physics 135-2, Final Exam
Fall 2016 (10 am Lecture)

Name: ______________________________

Discussion Section: __________________

1) The battery in your car is rated at 12.6 volts. When the headlights in your car are on (without the motor running), they produce 508 watts of energy. However, while you are starting the car, they "pull down" and produce only 225.8 watts of energy. (The resistance of the headlights is 0.2 Ω.)

1a) (5 points) What is the internal resistance of the battery? You may assume a simple series circuit for the headlights and the battery.

Solution
When the motor is off, the current through the headlights is given by $P = I^2R$, or in this case, by $508 = I^2(0.2)$, which means I = 50.4 amps. The current through the battery will be given by $\mathcal{E} = (r + R)I$, where r is the internal resistance of the battery and R is the resistance of the headlights. We have: $r = \mathcal{E}/I - R = 12.6 / 50.4 - 0.2 =$ 0.05 Ω

1b) (5 points) If the resistance of the electric starting motor is 0.08 Ω, and it is placed into parallel with the headlights when it is turned on, what current does it pull?

Solution
The resistance R of the motor plus the lights is given by: $(^1/_{0.2} + {^1/_{0.08}})^{-1} = 0.05714\ \Omega$. The total resistance of the circuit is then 0.05714 Ω *plus* the internal resistance of the battery, or 0.10714 Ω. Therefore the current flowing in the circuit is 12.6 / 0.10714 = 117.6 amps. The voltage drop across the internal resistance of the battery will be (117.6)(0.05) = 5.88 volts, which means the voltage across the motor/light will be 6.72 volts. Therefore the motor will be drawing 6.72 / 0.08 = 84 amps.

An alternative way to solve this problem is to realize that when two resistors R_1 and R_2 are placed in parallel, we have $V = I_1R_1 = I_2R_2$, i.e., we have a current flowing through R_1 that is R_2/R_1 times the current flowing through R_2. In this case, I_2 is $225.8 = I_2^2(0.2)$, or $I_2 = 33.6$ amps, so $I_1 = (33.6)(0.2 / 0.08) =$ 84 amps, as before.

2) (10 points) You have a very thin, conducting circle of hot plasma gas which initially has a radius of r = 2 cm. The circle of gas is sitting in a uniform magnetic field of 0.5 T with its normal oriented at 30° to the field. Then, the magnetic field begins increasing at a constant rate of dB/dt = 0.10 T / s, *and* the circle begins expanding at a rate of dr/dt = 2 cm/s. What will be the magnitude of the emf in the circle two seconds after it starts expanding?

Solution

From Faraday's law of induction, we have $\mathcal{E} = d\Phi/dt$. In this case $\Phi = BA\cos\theta$ means that we will have $\Phi = B(\pi r^2)\cos(30°)$ at any given instant. To find $d\Phi/dt$ we just use the chain rule of calculus and obtain $d\Phi/dt = (dB/dt)(\pi r^2)\cos(30°) + B\cos(30°)(2\pi r)(dr/dt)$.

At t = 2 seconds, r will have increased to 6 cm, and B will have increased to 0.7 T, so
$\mathcal{E} = (0.1)\pi(0.06)^2\cos(30°) + (0.7)\cos(30°)(2\pi)(0.06)(0.02) =$ 5.55 mV

3) (10 points) A long coil of wire is connected in series with a 10 kΩ resistor. An ideal 50-volt battery is suddenly connected across both devices, and 5.00 ms later there is 2.00 mA of current flowing in the resistor. How much energy is stored in the coil at this same moment?

Solution: We know that the energy in the coil will be $U = ½LI^2$, but we do not know L. However, we do know that $I(t) = (\mathcal{E}/R)[1 – \exp(–tR/L)]$. Inverting this equation gives us: $\ln[1 – (IR/\mathcal{E})] / (–tR) = 1/L$, or $L = –(5 \times 10^{-3})(10{,}000)/ \ln[1 – (2 \times 10^{-3})(10{,}000)/50] =$ 97.88 H.

$U = ½(97.88)(2 \times 10^{-3})^2 =$ 1.96×10^{-4} J

4) An ac generator operating at $\mathcal{E}_{max}$ = 220 V and f = 400 Hz causes oscillations in a series circuit consisting of a resistor with R = 220 Ω, a capacitor with C = 24.0 μF, and an inductor with L = 150 mH.

4a) (6 points) What is the maximum current at any time in this circuit?

Solution
We have $X_C = 1/\omega C = 1/(2\pi)(400)(24 \times 10^{-6}) = 16.6\ \Omega$.
We have $X_L = \omega L = 2\pi(400)(0.15) = 377\ \Omega$.

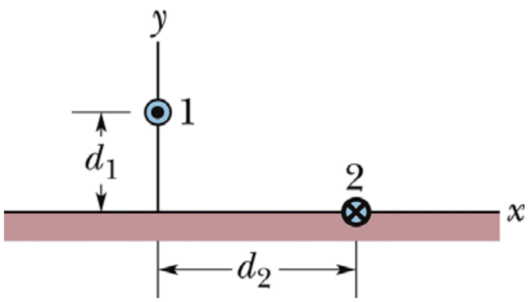
This gives us an impedance of $Z^2 = 220^2 + (377 - 16.6)^2$, or Z = 422.2 Ω

The maximum current is then $\mathcal{E}$ = IZ, or I = 220/422.2 = 0.521 amp

4b) (4 points) If a second capacitor with the same capacitance as the first is connected in series with the first, what will be the new impedance of the circuit?

Solution
The equivalent capacitance will drop to half of what it was, so X_C will double. We have $Z^2 = 220^2 + (377 - 33.2)^2$, or Z = 408 Ω

5a) (7 points) Two very long wires are carrying currents. Wire 1 is carrying current out of the page, with i_1 = 3 amps and d_1 = 2 cm. Wire 2 is carrying current into the page, with i_2 = 6 amps and d_2 = 3 cm. What is the magnitude of the combined magnetic field due to these two wires at the origin of the diagram? (i.e., at y = 0, x = 0)



Solution
The magnitudes of the magnetic fields from each wire will be given by $B = \mu_0 I/2\pi r$. This quickly yields: $B_1 = (2 \times 10^{-7})(3)/0.02 = 3 \times 10^{-5}$ T, and $B_2 = (2 \times 10^{-7})(6)/0.03 = 4 \times 10^{-5}$ T.

The magnetic fields due to each wire will be at 90° to each other at the origin. This therefore means we can find their combined magnitude by using the Pythagorean Theorem. We have $B = (3^2 + 4^2)^{1/2} \times 10^{-5}$ T = 5×10^{-5} T

5b) (3 points) In what direction is the magnetic field at the origin pointing? Assume that $\theta = 0$ is given by the positive x-axis.

Solution
Using the right-hand rule, we see that the magnetic field due to wire 1 will be pointing along the positive x-axis. Likewise, the right-hand rule tells us that the magnetic field due to wire 2 will be pointing along the positive y-axis. We know that $\tan\theta$ = opposite/adjacent, or in this case, $\tan\theta = (4 \times 10^{-5}) / (3 \times 10^{-5})$, or $\theta = 53.13°$

6) (10 points) Two infinite conducting flat plates are separated by a distance of a = 5 cm. There exists an electric potential between the plates given by $V(x) = V_0(x/a)^{4/3}$, where V_0 = 500 volts and a = 5 cm. (In other words, assume one plate is at x = 0 and the other plate is at x = 5 cm.) If you place an electron at x = 2 cm, what will be the magnitude of the acceleration acting on it? The mass of the electron is 9.11×10^{-31} kg.

Solution

We have F = ma = qE for the force on the electron. We are not given the electric field, but we know that E = dV/dx, so $E(x) = V_0(^4/_3)x^{1/3} / a^{4/3} = (500)(1.33)(0.02^{1/3}) / 0.05^{4/3} =$ 9824 volts/m at x = 2 cm.

Then $a = (1.6 \times 10^{-19})(9824)/(9.11 \times 10^{-31}) = 1.73 \times 10^{15}$ m/s^2

7) (10 points) Using the fact that $f = qB/2\pi m$ for a cyclotron, estimate the total path length traveled by a deuteron in a cyclotron of radius 50 cm and operating frequency 10 MHz during the (entire) acceleration process. Assume that the accelerating potential between the dees is 80 kV. The deuteron is a particle which has a charge of +e and a mass of 3.346×10^{-27} kg.

Solution

From the given numbers, we see that the magnetic field operating in this particular cyclotron is $B = 2\pi fm/q = (2\pi)(10 \times 10^6)(3.346 \times 10^{-27})/(1.6 \times 10^{-19}) = 1.31$ T.

Next, we know from $qvB = mv^2/r$ that $v = qBr/m$, so $E = \frac{1}{2} mv^2$ means that the maximum energy of a deuteron in the cyclotron is given by $E = e^2B^2r^2/2m$. We can save a little time and convert this into eV by dividing through by e, so the maximum energy in eV then becomes $E = (1.6 \times 10^{-19})(1.31)^2(0.50)^2/2(3.346 \times 10^{-27}) = 10.26$ MeV.

The deuteron will gain 2 x 80 keV during each cycle it makes around the cyclotron, so the total number of cycles it will make is $(10.26 \times 10^6)/2(80 \times 10^3) = 64$ cycles.

If we take the average circumference of an orbit in the cyclotron to be $2\pi(0.50/2) = 1.57$ m, then we end up with a total distance travelled of (1.57)(64) = 101 m.

Alternatively, you could say that the average energy of the deuteron is 5.13 MeV, which means that its average velocity is $(5.13 \times 10^6)(1.6 \times 10^{-19}) = \frac{1}{2}(3.346 \times 10^{-27})v^2$, or $v = 2.2 \times 10^7$ m/s. 64 cycles at a period of 1/f means that the deuteron is in flight for $64/(10 \times 10^6)$ seconds, which means it will travel $d = vt = (2.2 \times 10^7)(64)/(10 \times 10^6) = 142$ m.

So, roughly, the deuteron will travel about 100 to 150 meters.

Multiple Choice. Select the one best answer.

_____ **8) (2 points)** Unpolarized light with an intensity of 12 W / m^2 is shining on a polarizing filter. A second polarizing filter is directly behind the first one, and is oriented at 30° relative to the first one. What intensity of light is emerging from the second filter?

A) 10.4 W / m^2 **B)** 9.0 W / m^2 **C)** 5.2 W / m^2
D) 4.5 W / m^2 **E)** 12 W / m^2 **F)** 6.0 W / m^2

Solution: D
The first filter will reduce the intensity by ½, because unpolarized light shining on a polarizing filter is always reduced by a half. The second filter will further reduce the intensity by a factor of $\cos^2\theta = \cos^2(30°) = 0.75$, so the final intensity is (12)(0.5)(0.75) = 4.5 W / m^2.

9) (1 point per correct answer) For each item listed below, indicate whether it is a:

A) ferromagnetic material
B) paramagnetic material
C) diamagnetic material
D) nonmagnetic material

_____ **9a)** an unmagnetized bar of soft iron
Answer: A. Iron is a ferromagnetic material, whether it is actually magnetized or not.

_____ **9b)** a vial of liquid oxygen
Answer: B. I went to some length in class to create liquid oxygen in a glass tube and then show that it is attracted by a strong magnet. However, the liquid oxygen can never be permanently magnetized, so that means it is a paramagnet.

_____ **9c)** an aluminum-nickel-cobalt permanent magnet
Answer: A. If it can be permanently magnetized, then it is a ferromagnet.

_____ **9d)** a lump of coal
Answer: C. Many students answered D for this one, but in fact, D is not the correct answer for anything. *All* materials are diamagnetic to some extent.

_____ **10) (2 points)** The so-called "spin" of an electron is physics jargon which refers to the fact that:

A) The intrinsic angular momentum of the electron never changes.
B) Electrons spin on tiny orbits around the atomic nucleus.
C) The magnetic dipole moment of the electron oscillates as $\sin^2(\omega t)$.
D) Electrons are like small spinning spheres.
E) Electrons are like small clouds, but can spin slowly.
F) The electron has a magnetic moment, thus it must spin.

Answer: A. The other answers are essentially nonsense.

______ **11) (2 points)** A small transformer in your computer converts 115 volts into 6 volts. If the primary transformer (the one connected to the 115 volts) has 1000 turns of wire in it, how many turns does the secondary transformer have?

A) 19,200	**B)** 1000	**C)** 19
D) 52,000	**E)** 115	**F)** 52

Solution: F

The proper formula is $N_1(V_2/V_1) = N_2$, so $N_2 = (1000)(6/115) = 52$.