Physics 135-2, Midterm 2, 1pm

Name: ______________________________

Discussion Section: __________________

1) A 12-volt ideal battery, which is turned off, is connected in series to a resistor of unknown size and a capacitor of 20 μF. Then the battery is turned on. After 0.30 seconds have passed, the voltage across the capacitor is 8.5 v.

1a) (5 points) What is the resistance of the resistor?

Solution

We have $V(t) = V_o(1 - e^{-t/RC})$. Inverting this equation for the resistance gives us:

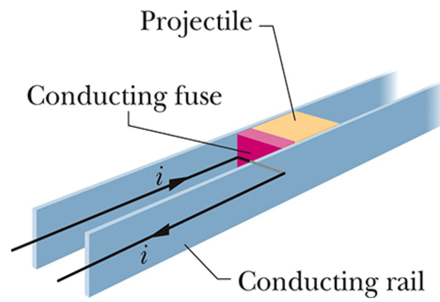
$R = -t / C \ln[-V(t)/V_o + 1] = (-0.3) / (20\ \mu F)\ln(1 - 8.5/12) =$ 12,174 Ω.

1b) (5 points) When the voltage across the capacitor reaches 10 v, the battery is suddenly turned off. What will be the voltage across the capacitor 0.30 seconds later?

Solution

We have $V(t) = Vo\ e^{-t/RC} = (10\ v)\exp[-(0.3\ s)/(12{,}174\ \Omega)(20\ \mu F)] =$ 2.92 volts.

2) (10 points) You have a rail gun which is set up with the rails 1 cm apart. A projectile of 25 g mass is attached to a fuse of negligible mass. Then, you use a large capacitor to send 2,000 amps through the rail gun. *Estimate* the acceleration of the projectile. Show all work and explain your reasoning to gain possible partial credit. (Hints – It is OK to assume that the rails are actually wires. You don't need to set up any integrals. This is an *estimate*.)



Solution

As indicated by the hints, the rails as shown in the figure are not exactly wires – but for the purposes of doing a mere estimate, we will use the formula for the magnetic field around a wire anyway. We thus have $B = \mu_0 i/2\pi r$ for the magnetic field of one rail. However, this is for an infinite wire (whereas our picture shows that the rails are only half-infinite), so we will take the field for one wire/rail to be $\mu_0 i/4\pi r$.

At this point we could get very complicated and write down the B field for each wire and try to integrate the $1/r$ functions across the fuse – but that defeats the whole purpose of just doing an estimate. Instead, let's simply calculate the B field at a point half-way across the fuse and take that to be our (constant) B field. This isn't a bad estimate, because as one moves away from the midpoint, the B field for one rail will decrease while that of the other rail increases, so the the two effects somewhat cancel out.

Using the formula for a wire, and doubling it because we have two rails:

$B = (2)(4\pi \times 10^{-7})(2000) / 4\pi(0.5 \times 10^{-2}) = 0.08$ T

The force acting on the fuse will be given by $F = iLB = (2000)(0.01)(0.08) = 1.6$ N.

Therefore the acceleration of the mass is $a = F/m = 1.6 / (25 \times 10^{-3}) = 65$ m/s^2.

Note – A significant number of students tried to solve this problem by writing down the formula for the magnetic field between two parallel wires. However, that formula can only tell you the force acting *between* the rails, which is not relevant to the force acting on the fuse. We did not give any credit for these answers.

3) An electron with a kinetic energy of 2.5 keV is moving along the positive x-axis (i.e., it is moving from left to right). It enters a region in which a uniform electric field of magnitude 10 kV/m is directed along the negative y-axis (i.e., with positive charge at the top, negative charge at the bottom). A uniform magnetic field B is to be set up such that the electron will continue moving in a straight line along the x-axis. (The electron mass is 9.11×10^{-31} kg.)

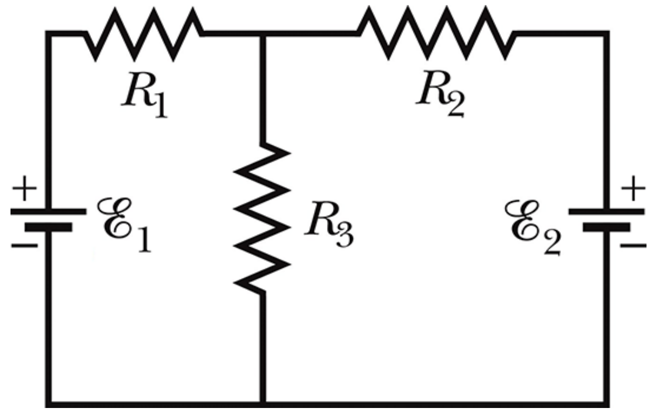
3a) (7 points) What must be the magnitude of the B field?

Solution
We require that the electric force on the electron equal the magnetic force, or eE = evB. This means B = E / v. We have $K = ½ mv^2$ for the kinetic energy of the electron, or $v = (2K/m)^{1/2} = [(2)(2500 \times 1.6 \times 10^{-19})/(9.11 \times 10^{-31})]^{1/2} = 2.96 \times 10^7$ m/s. This yields
$B = 10{,}000 / 2.96 \times 10^7 = 3.38 \times 10^{-4}$ T.

3b) (3 points) Along what axis and in what direction along that axis must the B field be oriented?

Reasoning: The electric field will cause the electron to bend "up", i.e., towards the positive ***j*** axis. The magnetic *force* must therefore be "down", along the negative ***j*** axis. But the electron is a negatively charged particle, so your thumb must point "up" in this case to use the right-hand rule on it. That means the magnetic field must be going "into the page", or along the negative ***k*** axis.

4) (10 points) In the circuit at right, we have $\mathcal{E}_1 = 16$ v, $\mathcal{E}_2 = 12$ v, $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$ and $R_3 = 20\ \Omega$. What is the voltage drop across R_2?



Solution

Since this is a multi-loop circuit, we will need to set up equations to solve for the current in all parts of the circuit. Simply stating that $\mathcal{E}_2 = i(R_2 + R_3)$ is not correct.

There are many ways to set up the equations. I will assume that we have one current flowing in a clockwise direction in the left-hand loop, and a second current flowing in a counter-clockwise direction in the right-hand loop. With these assumptions, we have:

$16 - i_1(5) - (i_1 + i_2)(20) = 0$ and $12 - i_2(10) - (i_1 + i_2)(20) = 0$

Subtracting the second equation from the first gives us:
$4 - 5i_1 + 10i_2 = 0$, or $0.8 + 2i_2 = i_1$

Substituting this result for i_1 back into the first equation yields:
$16 - 5(0.8 + 2i_2) - 20(0.8 + 2i_2 + i_2) = 0$, or
$16 - 4 - 10i_2 - 16 - 60i_2 = 0$, or $i_2 = -2/35$ amp $= -0.057$ amp.

The negative sign indicates that the current in R_2 is flowing clockwise, not counter-clockwise as we originally assumed. However, since we are only interested in the voltage drop, the minus sign is not important. From $V = iR$, we have $V = (2/35)(10) = 0.571$ volt.