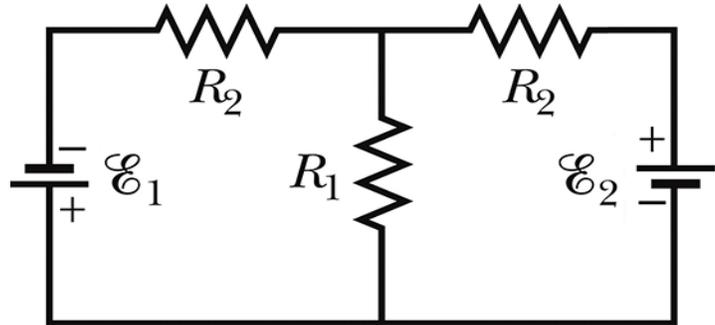1) (10 points) The circuit shown at right has $\mathcal{E}_1$ = 9 v, $\mathcal{E}_2$ = 6 v, R_1 = 5 Ω, and both R_2 = 2 Ω. How much heat (power) is being disipated by resistor R_1?

Solution

First we must determine the currents flowing in the circuit. Let us assume that we have two: one flowing counter-clockwise in the left-hand loop, and one flowing counter-clockwise in the right-hand loop. We can then write down loop equations for each side:

$9 - (i_1 - i_2)(5) - i_1(2) = 0$ and $6 - i_2(2) - (i_2 - i_1)(5) = 0$

There are many ways to do the algebra at this point. If we simply add the two equations together we get: $15 - 2i_1 - 2i_2 = 0$, or $i_1 = 7.5 - i_2$. Substituting this back into the second equation yields $6 - 2i_2 - 5(i_2 - 7.5 + i_2) = 0$, or $6 - 2i_2 - 10i_2 + 37.5 = 0$, or $i_2 =3.625$ amps. This then yields $i_1 = 3.875$ amps.

From the way that we chose our initial currents, we can see that i_1 and i_2 are flowing in different directions in R_1. Thus the net current through R_1 is 3.875 – 3.625 = 0.25 amps. This gives us a power in R_1 of P = i^2R = $(0.25)^2(5)$ = 0.3125 watts.

2a) (3 points) Suppose I have some gold wire which has a radius of 0.5 mm and a length of 200 m. If the resistivity of gold is 2.35 x 10^{-8} Ω m, what is the resistance of the wire?

Solution

R = ρ L / A = (2.35 x 10^{-8})(200)/π(0.5 x 10^{-3})² = 5.98 Ω

2b) (2 points) Suppose I now pass the wire through a roller which squishes the wire into a sheet of gold foil that is only 10^{-6} m thick. (The length of the finished roll of foil is still 200 m.) Will the resistance of the foil along its length be (circle one):

A) 500 times greater than the original length of wire

B) 78.5% greater than the original length of wire

C) The same as the original length of wire

D) 0.785 times less than the original length of wire

E) 500 times less than the original length of wire

Solution
C) Because mass is conserved, the cross-sectional area of the gold will not change as its shape is changed from circular to flat. Thus the resistance will not change.

3) (5 points) It is 1923, and you are living in a house which runs off a windmill-powered electrical system that generates a constant 48 volts of DC power. You have an old space heater which puts out 1000 watts of heat when it is operating normally. Then, it breaks down. You open it up and discover that that it has 1.44 meters of Nichrome wire inside it, 20 cm of which has become so corroded that it no longer conducts. You fix the space heater by just cutting out the corroded wire and splicing the rest of the wire back together. How much heat will the repaired space heater now generate?

Solution: By using $P = V^2 / R$, we can see that the wire in the space heater originally had a resistance of $R = V^2 / P = 48^2 / 1000 = 2.3 \ \Omega$. Afterwards, since the resistance of the wire is proportional to its length, the resistance will drop to 2.3 (1.44 – 0.2) / 1.44 = 1.98 Ω. The power generated by the repaired heater will then be $P = 48^2 / 1.98 = $ 1160 watts.

4) (10 points) A circular coil of wire with a radius of 8.0 cm is carrying a current of 0.20 amp. The coil has 10 loops of wire in it, and is oriented such that a normal to the circle points along n = 0.60 i – 0.80 j. If a magnetic field of B = 0.25 i + 0.30 k (the units are in Tesla) is placed across the coil, what will be the magnitude of the torque operating on the loop?

**Solution:** We can use the relation $\tau = \mu \times B$. The magnitude of μ = NiA, so we can replace the given normal vector with $(10)(0.2)\pi(0.08)^2$ n = 0.04 n = 0.024 i + 0.032 j. Then, using the traditional matrix method of computing a vector cross-product, we have:

i	j	k
0.024	0.032	0
0.25	0	0.30

τ = (0.032)(0.30) i – (0.024)(0.30) j – (0.25)(0.032) k

The magnitude of this is $[(0.032)^2(0.30)^2 + (0.024)^2(0.30)^2 + (0.25)^2(0.032)^2]^{1/2}$ = 0.014 N m.

5a) **(9 points)** A very long wire carrying a current of 5.7 amps is bent into a circular section with a radius of 1.9 cm, as shown at right. What is the magnitude of the magnetic field at the center C of the circle?
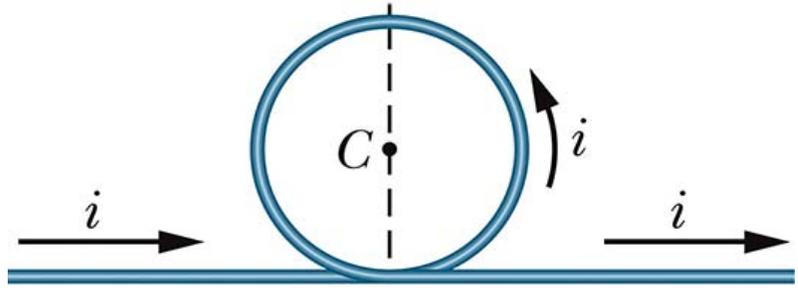
Solution: There are essentially two magnetic fields here, one due to the long straight section of the wire, and one due to the circular section of wire. Because of the symmetry of the problem, the fields will just add. The relevant formulas are:

B(long wire) = μ_o i / 2πR
B(full circle) = μ_o i / 2R

Adding them yields (μ_oi/2R)(1 + $^1/_\pi$) = (4π x 10^{-7})(5.7)(1.3183)/2(0.019) = 2.48 x 10^{-4} T

5b) **(1 point)** In what direction is the magnetic field at C pointing? __E__

  A) To the left.
  B) To the right.
  C) Directly up (along dotted line).
  D) Directly down (along dotted line).
  E) Out of the page.
  F) Into the page.