Phyx 135-2, Midterm I (1pm), Fall 2016 **Name: ______________________________**

Discussion Section: ____________________

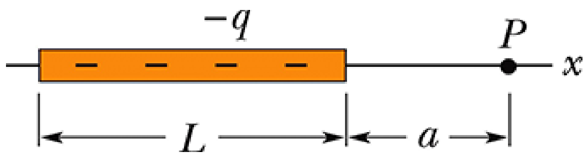
1) (10 points) Suppose a charge of $Q_1 = -5.00e$ is placed at x = 0 on an x-axis, and a charge of $Q_2 = +2.00e$ is placed at x = 3 cm on that same axis. If an unknown charge Q_3 is to be located such that the net electrostatic force on it from charges Q_1 and Q_2 is zero, what must be the x-coordinate of charge Q_3?

Solution

We are looking for a spot where the forces from Q_1 and Q_2 cancel. Since the two charges are not equal, this spot must be closer to Q_2 than to Q_1 so that the $1/r^2$ factor in Coulomb's Law can come into play and "boost" the force from Q_2. Also, we see that this spot must be to the *right* of Q_2, because to the left of Q_2 the forces from the charges act in the same direction and cannot cancel.

So, we are looking for the coordinate x where $k(5e) / x^2 = k(2e) / (x - 0.03)^2$. A bit of algebra yields: $0.2\,x^2 = 0.5(x^2 - 0.06x + 9 \times 10^{-4})$, or $0.3\,x^2 - 0.03x + 4.5 \times 10^{-4} = 0$, or $x^2 - 0.1x + 1.5 \times 10^{-3} = 0$. Using the quadratic equation, $2x = 0.1 \pm (0.01 - 6 \times 10^{-3})^{1/2}$, and we see that x can equal either 8.16 cm or 1.84 cm. We must reject the smaller solution because that would place Q_3 between Q_1 and Q_2, and thus x = 8.16 cm.

2) (10 points) Suppose you have a nonconducting rod of length L with a charge of –q distributed evenly along it. Derive the electric field due to the rod at a distance of "a" from the right-hand side of the rod. (Show all work. Writing down a memorized equation will not receive credit.)



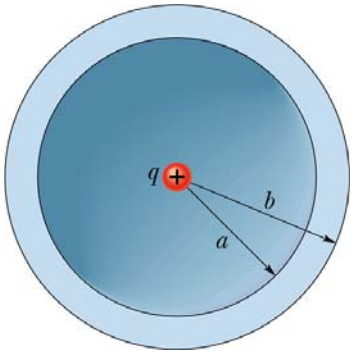
Solution
The most direct way to solve this problem is to integrate the electric field. We know that we can divide the rod into an infinite number of dq's, with $dE = k\, dq / r^2$. From symmetry we know that we are working in one dimension, so the direction of the electric field is just along the x-axis. This in turn means that ***r*** (or "r-hat") is always one, so we can ignore it.

We thus have: $dE = k\, dq / r^2 = k\, (-q/L)\, dx / x^2$, where we have used the standard "trick" of substituting $dq = \lambda\, dx$. Integrating gives us $E = kq / Lx$. We must integrate from $x = a$ to $r = L + a$, so we have: $E = (kq/L)[1/(L + a) – 1/a] = -kq / a(L + a)$.

Alternatively, one could integrate the electric potential and then take a derivative. This gives us:

$dV = k\, dq / r = k(-q/L)\, dx / x$, or $V = -(kq/L) \ln x$. Substitution of the limits yields:
$V = -(kq/L)[\ln(L + a) – \ln a]$, and taking negative the derivative with respect to "a" puts us right back at the expression for E given above.

3) A nonconducting spherical shell of inner radius a = 2 cm and outer radius b = 2.4 cm carries a uniform charge density (only within the shell itself) of $\rho = 60$ nC / m^3. In addition, there is a point charge of q = 2 pC at the center of the shell.



a) (5 points) What is the magnitude of the electric field at r = 3 cm?
b) (5 points) What is the magnitude of the electric field at r = 2.3 cm?

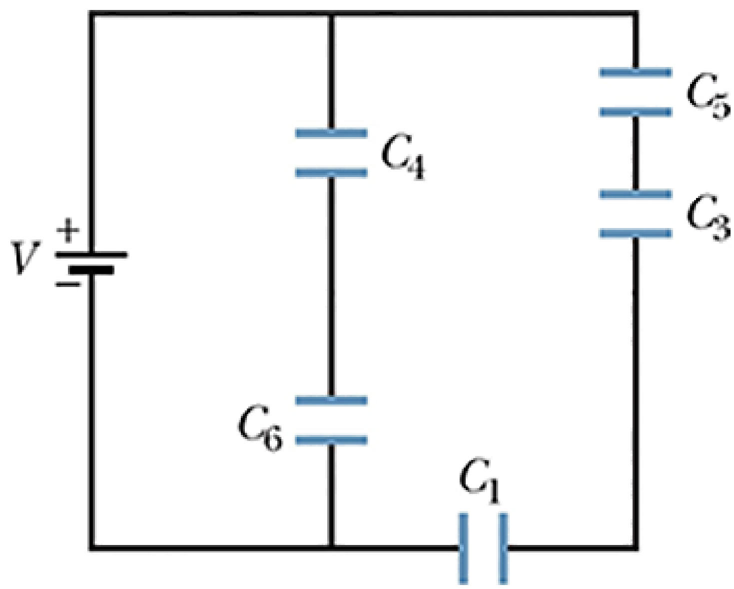
Solutions
By using Gauss' Law and invoking symmetry (as discussed in class several times), we can write down $\varepsilon_o E(4\pi r^2) = q_{enc}$ for this geometry. For Part a, we are completely outside the shell, so the total charge of the shell is enclosed. This is $Q = \rho V = \rho(4\pi/3)b^3 - \rho(4\pi/3)a^3 = (4\pi\rho/3)(b^3 - a^3) =$ $(4.18879)(60 \times 10^{-9})(0.024^3 - 0.02^3) = 1.464 \times 10^{-12}$ C = 1.464 pC.

Including the charge at the center gives us 3.464 pC enclosed. We then have:
$\varepsilon_o E(4\pi r^2) = Er^2 / k = 3.464$ pC, or $E = (3.464 \times 10^{-12})(8.99 \times 10^9) / 0.03^2 =$ 34.6 volts/m.

For Part b, we have similar logic, but different numbers. The charge enclosed in the shell is:
$Q = (4.18879)(60 \times 10^{-9})(0.023^3 - 0.02^3) = 1.047 \times 10^{-12}$ C, so the total charge enclosed by the Gaussian surface is 3.047 pC.

This yields $E = (3.047 \times 10^{-12})(8.99 \times 10^9) / 0.023^2 =$ 51.8 volts/m.

4) (10 points) In the figure at right, suppose that all of the capacitors have a capacitance of 5 μF. If the battery is supplying V = 9 volts, how much charge is this network holding?



Solution

C_1, C_3, and C_5 are in series, so their net capacitance C is given by:

$1/C = {}^{1}/_{5} + {}^{1}/_{5} + {}^{1}/_{5}$ in μF, or

$C = {}^{5}/_{3}$ μF.

Likewise, the net capacitance of C_4 and C_6 is $1/C = {}^{1}/_{5} + {}^{1}/_{5}$, or $C = {}^{5}/_{2}$ mF.

These two net capacitances are in parallel, so their capacitances will simply add. The net capacitance of the circuit is thus ${}^{5}/_{3} + {}^{5}/_{2} = {}^{25}/_{6}$ μF.

The charge in the network will be $q = CV = ({}^{25}/_{6}$ μF)(9 volts) $= {}^{75}/_{2}$ μC = 37.5 μC.