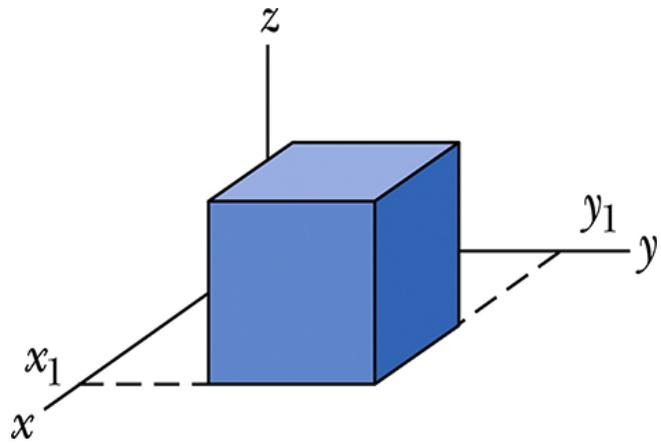


1) (5 points) A dipole with $q = 0.1 \text{ mC}$ is placed in a uniform electric field of 23 N/C . The electric field is oriented along the x -axis; the dipole is oriented at $\theta = 22^\circ$ to the x -axis. The dipole is then turned to $\theta = 66^\circ$, and you are informed that it took 7 mJ of energy to do this. How far apart are the charges in the dipole?

Solution: We have $U = -pE \cos\theta$ for a dipole in a uniform field. Inserting numbers, we have $\Delta U = 7 \text{ mJ} = -pE[\cos(66^\circ) - \cos(22^\circ)] = -p(23)(0.4067 - 0.9272) = 11.97 \text{ p}$. But $p = qd$, so $7 \text{ mJ} = (11.97)(0.1 \text{ mC})d$, which gives $d = 7 / 1.197 = 5.85 \text{ meters}$.

2) (10 points) The figure shows a Gaussian surface in the shape of a perfect cube with an edge length of 2.00 m. One corner is at $x_1 = 5.00$ m and $y_1 = 4.00$ m. There is an electric field running through the cube given by $\mathbf{E} = -3 \mathbf{i} - 4y^2 \mathbf{j} + 3 \mathbf{k}$ N/C, with y in meters. What is the net charge contained by the cube?

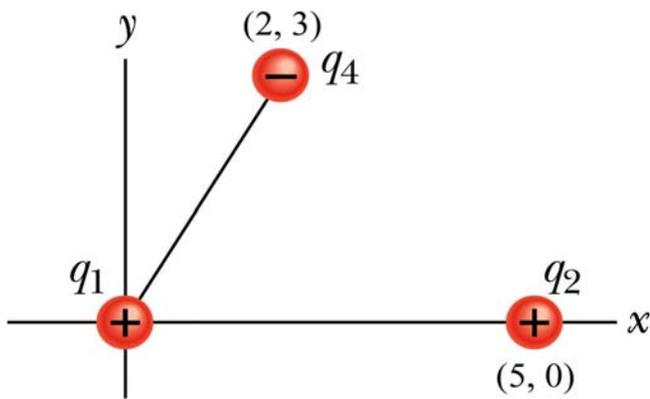


Solution

Gauss's Law says that we must add up the electric flux through the six sides of the cube to obtain the net charge inside. We notice immediately that for the two sides facing along the x -axis, we have $\Phi = \mathbf{E} \cdot \mathbf{A} = \pm 3A$, where A is the area of one side of the cube. In other words, because they are facing in opposite directions, one side gives you $+3A$ and the other side gives you $-3A$, so they cancel out.

Likewise, the "top" and "bottom" of the cube give the same flux except with opposite signs, so they also cancel out.

Now, along the \mathbf{j} axis we will have $-4y^2A$ for the right-hand side of the cube, with $y = 4$. For the left-hand side of the cube we will have $+4y^2A$ with $y = 2$. Inserting numbers gives us: $-4(4^2)(4) + 4(2^2)(4) = -192$. The charge enclosed is thus $-192\epsilon_0 = -1.7 \times 10^{-9}$ C.



3) (10 points) Three charges of $q_1 = +1$ C, $q_2 = +2$ C, and $q_4 = -4$ C are arranged as shown. The numbers in parentheses represent their x, y coordinates in meters. What is the force vector operating on q_4 ?

Solution: The distance from q_1 to q_4 is $(2^2 + 3^2)^{1/2} = \sqrt{13}$. Therefore the force vector operating between those two charges is $F = k(+1)(-4)/13(2\mathbf{i} + 3\mathbf{j})/\sqrt{13}$, where $(2\mathbf{i} + 3\mathbf{j})/\sqrt{13}$ is the unit vector pointing from q_1 to q_4 . This gives $F = (-4k/13^{3/2})(2\mathbf{i} + 3\mathbf{j})$.

The distance from q_2 to q_4 is $(3^2 + 3^2)^{1/2} = \sqrt{18}$. Therefore in a similar fashion the force vector operating between those two charges is $F = (-8k/18^{3/2})(-3\mathbf{i} + 3\mathbf{j})$.

Combining the vectors yields $F = -k [(0.1707 - 0.3143)\mathbf{i} + (0.2560 + 0.3143)\mathbf{j}] = (9 \times 10^9)(0.1436\mathbf{i} - 0.5703\mathbf{j})$ newtons.

4a) (3 points) A hollow metal shell of radius 20 cm has a total charge on its surface of 6×10^{-9} C. What is the electric potential V (in volts) at the center of the sphere? (We assume that $V = 0$ at infinity.)

Solution: The electric potential at the surface of the sphere is given by $V = kq/r = (9 \times 10^9)(6 \times 10^{-9})/0.2 = 270$ volts. Because the electric potential everywhere on a conductor and in a conductor must be equal, the potential at the center of the sphere is also 270 volts.

4b) (2 points) If I connect a thin wire to the outside of the sphere in Part (4a) and raise the potential of the sphere to 350 volts relative to ground, what then is the electric potential V at the center of the sphere?

Solution: 350 volts

5) (10 points) You have two infinite non-conducting parallel plates which are a distance $d = 35$ cm apart. A uniform electric field of $E = 3$ volts/meter runs between the plates *from left to right*. In addition, a positive charge density of $\sigma = 0.04$ nC / m² exists on the surface of the left-hand plate. If an electron is placed 10 cm to the right of the left-hand plate, what energy will it have (in eV) when it impacts one of the plates?

Solution: There are two electric fields here. The one due to the surface charge is given by: $E = \sigma/2\epsilon_0 = (4 \times 10^{-11})/2(8.85 \times 10^{-12}) = 2.26$ volts/meter. Since the charge is positive, the electric field will extend *away* from the charge, i.e., from left to right. This yields a total electric field (running left to right) of $3 + 2.26 = 5.26$ volts/meter between the plates. Since the electron is 0.1 meter from the positive plate, it will move towards the positive plate and gain **0.526 eV** of energy.