

1) A hydrogen atom can be thought of as a uniform ball of positive charge (the proton) surrounded by a uniform spherical shell of negative charge (the electron). Assume that the proton has a radius of 10^{-15} m and that the electron shell has a radius of 5×10^{-11} m.

Useful facts: $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N m}^2$, $e = 1.6 \times 10^{-19} \text{ C}$

a) (5 points) What is the charge density of the proton?

Solution: The charge on the proton is $e = +1.6 \times 10^{-19} \text{ C}$. Therefore $\rho = e / \text{volume} = e / (4/3\pi r^3) = (1.6 \times 10^{-19}) / (1.333)(3.14159)(10^{-15})^3 = 3.8 \times 10^{25} \text{ C/m}^3$.

b) (15 points) What is the electric field at a radius of 5×10^{-16} m from the center of the proton?

Solution: This radius places the field well within the proton itself. From Gauss' Law, and using spherical symmetry, we know: (electric field) X (surface area of sphere) = (charge enclosed) / ϵ_0 .

So, $E(4\pi r^2) = \rho V / \epsilon_0 = \rho(4\pi/3)r^3 / \epsilon_0$, or $E = \rho r / 3\epsilon_0$. Inserting numbers,

$E = (3.8 \times 10^{25} \text{ C/m}^3)(5 \times 10^{-16} \text{ m}) / (3)(8.85 \times 10^{-12} \text{ C}^2 / \text{N m}^2) = 7.2 \times 10^{20} \text{ N/C}$.

