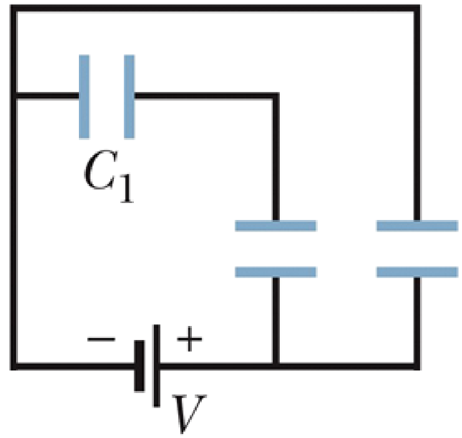
Phyx 135-2 Midterm #2 (10 am) **Name** ________________________________



1) In the network at right, C_1 = 6 μF and the other two capacitors have C = 3 μF. We have V = 9 volts.

1a) (5 points) What is the electrostatic energy contained in C_1?

1b) (5 points) Suppose I fill C_1 with a dielectric that has κ = 3.00 and fill the other two capacitors with a κ = 2.00 dielectric. (You may assume that they are all parallel plate capacitors.) What would be the electrostatic energy contained in C_1 then?

Solutions

1a) There are several ways to solve this problem. For example, the equivalent capacitance of C_1 plus the capacitor wired to it in series is given by: $1/C_{eq} = 1/(6 \times 10^{-6}) + 1/(3 \times 10^{-6})$, or C_{eq} = 2 μF. This means that the charge on the equivalent capacitor (and therefore, also the charge on C_1) is q = CV = (2 μF)(9 v) = 18 μC. We then have $U = ½\, q^2/C_1 = (0.5)(18 \times 10^{-6})^2 / (6 \times 10^{-6})$ = 2.7×10^{-5} J.

Another way to solve the problem is to realize that C_1 has double the capacitance of its series partner, therefore it must have half the voltage across it, or 3 v. (The partner has 6 v.) We then have $U = ½\, CV^2$ = $(0.5)(6\ \mu F)(3\ v)^2 = 2.7 \times 10^{-5}$ J.

1b) The capacitance of C_1 becomes 18 μF and the one wired to it in series becomes C = 6 μF. Their equivalent capacitance is then: $1/C_{eq} = 1/(18 \times 10^{-6}) + 1/(6 \times 10^{-6})$, or C_{eq} = 4.5 μF. Similarly to part (a), this gives us a charge on C_1 of q = (4.5 μF)(9 v) = 40.5 μC. We then have $U = ½\,(40.5 \times 10^{-6})^2 / (18 \times 10^{-6})$ = 4.56×10^{-5} J.

2) (5 points) Suppose we have a copper wire of radius 0.3 mm which is 1000 m long. And, suppose we have an aluminum wire of unknown radius that is also 1000 m long. What radius would the aluminum wire need to be if it is to have the same resistance as the copper wire?

resistivity of copper = $1.69 \times 10^{-8}\ \Omega$ m resistivity of aluminum = $2.75 \times 10^{-8}\ \Omega$ m

Solution

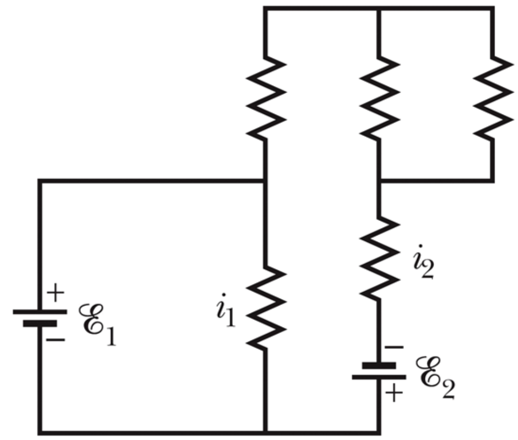
The resistance of the wires is given by $\rho L/A$. In this case we want the resistances to be equal, so we have $\rho_{Cu} L_{Cu} / A_{Cu} = \rho_{Al} L_{Al} / A_{Al}$, or $A_{Al} = (\rho_{Al} / \rho_{Cu})A_{Cu}$. The cross-sectional areas of the wires are proportional to πr^2, so we have $r_{Al} = (\rho_{Al} / \rho_{Cu})^{1/2} r_{Cu} = (2.75 / 1.69)^{1/2}$ (0.3 mm) = 0.383 mm.

3) (5 points) A parallel-plate capacitor is filled with a $\kappa = 5.5$ dielectric. The area of each plate is 0.034 m^2 and the plates are separated by 2.0 mm. The capacitor will fail (short out) if the electric field between the plates exceeds 200 kN/C. What is the maximum energy that the capacitor can store?

Solution

Since $V = Ed$, we see that the maximum voltage the capacitor can sustain is $(2 \times 10^5)(0.002) = 400$ v. We have $U = ½\ CV^2$ with $C = \kappa\varepsilon_0 A/d$, so $U = ½\ (5.5)(8.85 \times 10^{-12})(0.034)/(0.002) \times 400^2 = 6.6 \times 10^{-5}$ J.

4) (10 points) In the network at right, $\mathcal{E}_1 = 6$ v, $\mathcal{E}_2 = 9$ v, and all of the resistors are 5 Ω. What are the values of currents i_1 and i_2, and are they going up or down through their respective resistors?



Solution

The two resistors at top right can be replaced by one equivalent resistor. Its resistance is $^1/_R = {}^1/_5 + {}^1/_5$, or R = 2.5 Ω. Then this resistance can be placed in series with the resistors on either side of it, to give R = 12.5 Ω. In short, we can reduce the network to just two resistors (labelled by i_1 and i_2), which have resistances of 5 and 12.5 Ω, respectively.

We can then write down two loop equations. For the left loop, starting at the battery and going clockwise, we have: $6 - (i_1 - i_2)(5) = 0$. For the right loop, starting at the battery and going clockwise, we have: $9 - (i_2 - i_1)(5) - i_2(12.5) = 0$. There are many ways to do the algebra from here, but probably the simplest is just to add the two equations to get: $15 - 0 - i_2(12.5) = 0$, or $i_2 = 1.2$ amps. The positive sign means i_2 is going down. Going back to the first equation, we have: $6 - (i_1 - 1.2)(5) = 0$, or $i_1 = 2.4$ amps. The positive sign means i_1 is going down.

Therefore the i_1 (as defined on the diagram) is 2.4 amps down minus 1.2 amps up = 1.2 amps down.

5) Suppose you have a cyclotron which has a radius of 53 cm and is capable of maintaining a uniform magnetic field of 1.20 Tesla. The voltage difference between the two halves of the cyclotron is 80 kV. A radioactive source at the exact center of the cyclotron is injecting α-particles into it at approximately zero velocity. (The mass of the α-particle is 6.68×10^{-27} kg. It has a charge of 2e.)

5a) (5 points) Estimate the maximum energy (in eV) that the α-particles emerging from this cyclotron can achieve.

Solution

By equating mv^2/r (centrifugal force) to the Lorentz force, qvB, we know that v = qBr/m for any radius. The maximum velocity will occur at maximum radius, so $v_{max} = (2)(1.6 \times 10^{-19})(1.2)(0.53) / (6.68 \times 10^{-27}) = 3.05 \times 10^7$ m/s. This corresponds to $\frac{1}{2} (6.68 \times 10^{-27})(3.05 \times 10^7)^2 / (1.6 \times 10^{-19})$ = 19.4 MeV.

5b) (5 points) About how long does it take for the α-particle to achieve this energy inside the cyclotron?

Solution

We know that f(2πr) = v, where f is the frequency of revolution, so f(2πr) = qBr/m, or f = qB/2πm = $(3.2 \times 10^{-19})(1.2) / 2\pi (6.68 \times 10^{-27}) = 9.14 \times 10^6$ Hz. This corresponds to the α-particle taking $1 / 9.14 \times 10^6 = 1.09 \times 10^{-7}$ s to make one cycle. Now, the α-particle will gain 2 x 80 kV of energy each time it makes a cycle, so it needs about $(19.4 \times 10^6) / (160 \times 10^3)$ = 121 cycles to gain its full energy. The total time to achieve E_{max} is then $(1.09 \times 10^{-7}$ s)(121) = 1.32×10^{-5} s.