3. We use $\Phi = \vec{E}\cdot\vec{A}$, where $\vec{A} = A\hat{\text{j}} = (1.40\text{m})^2\,\hat{\text{j}}$.

(a) $\Phi = (6.00\ \text{N/C})\,\hat{\text{i}}\cdot(1.40\ \text{m})^2\,\hat{\text{j}} = 0.$

(b) $\Phi = (-2.00\ \text{N/C})\,\hat{\text{j}}\cdot(1.40\ \text{m})^2\,\hat{\text{j}} = -3.92\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \left[(-3.00\ \text{N/C})\,\hat{\text{i}} + (400\,\text{N/C})\,\hat{\text{k}}\right]\cdot(1.40\ \text{m})^2\,\hat{\text{j}} = 0$.

(d) The total flux of a uniform field through a closed surface is always zero.

4. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E.$ Thus, the flux through the netting is

$$\Phi' = -\Phi = -\pi a^2 E = -\pi(0.11\ \text{m})^2(3.0\times10^{-3}\ \text{N/C}) = -1.1\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C} .$$

12. We note that only the smaller shell contributes a (nonzero) field at the designated point, since the point is inside the radius of the large sphere (and E = 0 inside of a spherical charge), and the field points toward the $-x$ direction. Thus, with R = 0.020 m (the radius of the smaller shell), L = 0.10 m and x = 0.020 m, we obtain

$$\vec{E} = E(-\hat{\text{j}}) = -\frac{q}{4\pi\varepsilon_0 r^2}\,\hat{\text{j}} = -\frac{4\pi R^2\sigma_2}{4\pi\varepsilon_0(L-x)^2}\,\hat{\text{j}} = -\frac{R^2\sigma_2}{\varepsilon_0(L-x)^2}\,\hat{\text{j}}$$

$$= -\frac{(0.020\ \text{m})^2(4.0\times10^{-6}\,\text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.10\ \text{m}-0.020\ \text{m})^2}\,\hat{\text{j}} = \left(-2.8\times10^4\ \text{N/C}\right)\hat{\text{j}}.$$

16. The total electric flux through the cube is $\Phi = \oint \vec{E}\cdot d\vec{A}$. The net flux through the two faces parallel to the yz plane is

$$\Phi_{yz} = \iint\left[E_x(x=x_2) - E_x(x=x_1)\right]dydz = \int_{y_1=0}^{y_2=1}dy\int_{z_1=1}^{z_2=3}dz\left[10+2(4)-10-2(1)\right]$$

$$= 6\int_{y_1=0}^{y_2=1}dy\int_{z_1=1}^{z_2=3}dz = 6(1)(2) = 12.$$

Similarly, the net flux through the two faces parallel to the xz plane is

$$\Phi_{xz} = \iint\left[E_y(y=y_2) - E_y(y=y_1)\right]dxdz = \int_{x_1=1}^{x_2=4}dy\int_{z_1=1}^{z_2=3}dz[-3-(-3)] = 0\,,$$

and the net flux through the two faces parallel to the *xy* plane is

$$\Phi_{xy} = \iint \left[E_z(z=z_2) - E_z(z=z_1)\right] dxdy = \int_{x_1=1}^{x_2=4} dx \int_{y_1=0}^{y_2=1} dy\left(3b-b\right) = 2b(3)(1) = 6b.$$

Applying Gauss' law, we obtain

$$q_{\text{enc}} = \varepsilon_0\Phi = \varepsilon_0(\Phi_{xy} + \Phi_{xz} + \Phi_{yz}) = \varepsilon_0(6.00b + 0 + 12.0) = 24.0\varepsilon_0$$

which implies that $b = 2.00\ \text{N/C}\cdot\text{m}$.

57. (a) For $r < R$, $E = 0$ (see Eq. 23-16).

(b) For r slightly greater than R,

$$E_R = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-7}\ \text{C}\right)}{\left(0.250\text{m}\right)^2} = 2.88\times10^4\ \text{N/C}.$$

(c) For $r > R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = E_R\left(\frac{R}{r}\right)^2 = \left(2.88\times10^4\ \text{N/C}\right)\left(\frac{0.250\ \text{m}}{3.00\ \text{m}}\right)^2 = 200\,\text{N/C}.$$

58. From Gauss's law, we have

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\sigma\pi r^2}{\varepsilon_0} = \frac{(8.0\times10^{-9}\,\text{C/m}^2)\pi(0.050\ \text{m})^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 7.1\,\text{N}\cdot\text{m}^2/\text{C}\ .$$