29. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing effect. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

(a) We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q_{\text{enc}} = Q1 + Q_2 = -Q_1 = -3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{\text{enc}}$, or

$$E = \frac{q_{\text{enc}}}{2\pi\varepsilon_0 Lr} = \frac{-3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(20.0\times1.30\times10^{-3}\text{m})} = -0.214\ \text{N/C},$$

or $|E| = 0.214$ N/C.

(b) The negative sign in E indicates that the field points inward.

(c) Next, for $r = 5.00\ R_1$, the charge enclosed by the Gaussian surface is $q_{\text{enc}} = Q_1 = 3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{\text{enc}}$, or

$$E = \frac{q_{\text{enc}}}{2\pi\varepsilon_0 Lr} = \frac{3.40\times10^{-12}\ \text{C}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(11.0\ \text{m})(5.00\times1.30\times10^{-3}\text{m})} = 0.855\ \text{N/C}.$$

(d) The positive sign indicates that the field points outward.

(e) We consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod has charge Q_1, the inner surface of the shell must have charge $Q_{\text{in}} = -Q_1 = -3.40\times10^{-12}$ C.

(f) Since the shell is known to have total charge $Q_2 = -2.00Q_1$, it must have charge $Q_{\text{out}} = Q_2 - Q_{\text{in}} = -Q_1 = -3.40\times10^{-12}$ C on its outer surface.

30. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{2\lambda_1}{4\pi\varepsilon_0(x+L/2)} + \frac{2\lambda_2}{4\pi\varepsilon_0(x-L/2)}\ .$$

Setting this equal to zero and solving for x we find

$$x = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)\frac{L}{2} = \left(\frac{6.0\mu\text{C/m} - (-2.0\mu\text{C/m})}{6.0\mu\text{C/m} + (-2.0\mu\text{C/m})}\right)\frac{8.0\text{ cm}}{2} = 8.0\text{ cm}.$$

31. We denote the inner and outer cylinders with subscripts i and o, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\text{ m})} = 2.3\times10^6\text{ N/C}.$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

$$E(r = 8.0\text{ cm}) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\text{ C/m} - 7.0\times10^{-6}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\text{ m})} = -4.5\times10^5\text{ N/C},$$

or $|E(r = 8.0\text{ cm})| = 4.5\times10^5\text{ N/C}$.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

49. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r = 0$, the above equation implies $E = 0$.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\text{ C})}{2(2.00\times10^{-2}\text{ m})^2} = 5.62\times10^{-2}\text{ N/C}.$$

(c) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(2.00\times10^{-2}\,\text{m})^2} = 0.112\ \text{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(1.50\times2.00\times10^{-2}\,\text{m})^2} = 0.0499\ \text{N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0.$ Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E}\cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.$$

52. The field is zero for $0 \le r \le a$ as a result of Eq. 23-16. Thus,

(a) $E = 0$ at $r = 0$,

(b) $E = 0$ at $r = a/2.00$, and

(c) $E = 0$ at $r = a$.

For $a \le r \le b$ the enclosed charge q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{\text{enc}} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right).$$

Therefore, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$.

(d) For r = 1.50a, we have

$$E = \frac{\rho}{3\varepsilon_0}\frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right) = \frac{(1.84\times10^{-9}\text{ C/m}^3)(0.100\text{ m})}{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{2.375}{2.25}\right) = 7.32\text{ N/C}.$$

(e) For $r = b$ = 2.00a, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{4}\right) = \frac{(1.84\times10^{-9}\text{ C/m}^3)(0.100\text{ m})}{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{4}\right) = 12.1\text{ N/C}.$$

(f) For $r \ge b$ we have $E = q_{\text{total}} / 4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}.$$

Thus, for r = 3.00b = 6.00a, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(6.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{36}\right) = \frac{(1.84\times10^{-9}\text{ C/m}^3)(0.100\text{ m})}{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{36}\right) = 1.35\text{ N/C}.$$

53. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz\, \rho = 4\pi\int_0^R dr\ r^2\ \rho = Q.$$

Substituting the expression $\rho = \rho_s r/R$, with ρ_s= 14.1 pC/m^3, and performing the integration leads to

$$4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = Q$$

or

$$Q = \pi\rho_s R^3 = \pi(14.1\times10^{-12}\text{ C/m}^3)(0.0560\text{ m})^3 = 7.78\times10^{-15}\text{ C}.$$

(b) At r = 0, the electric field is zero (E = 0) since the enclosed charge is zero.

At a certain point within the sphere, at some distance r from the center, the field (see Eq. 23-8 through Eq. 23-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2}$$

where q_{enc} is given by an integral similar to that worked in part (a):

$$q_{\text{enc}} = 4\pi\int_0^r dr\, r^2\rho = 4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{r^4}{4}\right).$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^4}{Rr^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^2}{R}.$$

(c) For $r = R/2.00$, where $R = 5.60$ cm, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s(R/2.00)^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R}{4.00} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})}{4.00}$$
$$= 5.58\times10^{-3}\,\text{N/C}.$$

(d) For $r = R$, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R^2}{R} = \frac{\pi\rho_s R}{4\pi\varepsilon_0} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})$$
$$= 2.23\times10^{-2}\,\text{N/C}.$$

(e) The electric field strength as a function of r is depicted below: