

28. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{d+x} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{d+x}.$$

To find the total potential at P_1 , we integrate over the length of the rod and obtain:

$$\begin{aligned} V &= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\epsilon_0} \ln(d+x) \Big|_0^L = \frac{Q}{4\pi\epsilon_0 L} \ln\left(1 + \frac{L}{d}\right) \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(56.1 \times 10^{-15} \text{ C})}{0.12 \text{ m}} \ln\left(1 + \frac{0.12 \text{ m}}{0.025 \text{ m}}\right) = 7.39 \times 10^{-3} \text{ V}. \end{aligned}$$

30. The dipole potential is given by Eq. 24-30 (with $\theta = 90^\circ$ in this case)

$$V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} = \frac{p \cos 90^\circ}{4\pi\epsilon_0 r^2} = 0$$

since $\cos(90^\circ) = 0$. The potential due to the short arc is $q_1 / 4\pi\epsilon_0 r_1$ and that caused by the long arc is $q_2 / 4\pi\epsilon_0 r_2$. Since $q_1 = +2 \mu\text{C}$, $r_1 = 4.0 \text{ cm}$, $q_2 = -3 \mu\text{C}$, and $r_2 = 6.0 \text{ cm}$, the potentials of the arcs cancel. The result is zero.

35. We use Eq. 24-41:

$$\begin{aligned} E_x(x, y) &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x} \left((2.0 \text{ V/m}^2)x^2 - 3.0 \text{ V/m}^2 y^2 \right) = -2(2.0 \text{ V/m}^2)x; \\ E_y(x, y) &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y} \left((2.0 \text{ V/m}^2)x^2 - 3.0 \text{ V/m}^2 y^2 \right) = 2(3.0 \text{ V/m}^2)y. \end{aligned}$$

We evaluate at $x = 3.0 \text{ m}$ and $y = 2.0 \text{ m}$ to obtain

$$\vec{E} = (-12 \text{ V/m})\hat{i} + (12 \text{ V/m})\hat{j}.$$

36. We use Eq. 24-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\vec{E} = -\left(\frac{dV}{dx}\right)\hat{i} = -\frac{d}{dx}(1500x^2)\hat{i} = (-3000x)\hat{i} = (-3000 \text{ V/m}^2)(0.0130 \text{ m})\hat{i} = (-39 \text{ V/m})\hat{i}.$$

(a) Thus, the magnitude of the electric field is $E = 39 \text{ V/m}$.

(b) The direction of \vec{E} is $-\hat{i}$, or toward plate 1.

37. We apply Eq. 24-41:

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2$$

$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2$$

$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz$$

which, at $(x, y, z) = (3.00 \text{ m}, -2.00 \text{ m}, 4.00 \text{ m})$, gives

$$(E_x, E_y, E_z) = (64.0 \text{ V/m}, -96.0 \text{ V/m}, 96.0 \text{ V/m}).$$

The magnitude of the field is therefore

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150 \text{ V/m} = 150 \text{ N/C}.$$

44. The work done must equal the change in the electric potential energy. From Eq. 24-14 and Eq. 24-26, we find (with $r = 0.020 \text{ m}$)

$$W = \frac{(3e - 2e + 2e)(6e)}{4\pi\epsilon_0 r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(18)(1.60 \times 10^{-19} \text{ C})^2}{0.020 \text{ m}} = 2.1 \times 10^{-25} \text{ J}.$$

45. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\epsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\epsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle.

Conservation of energy yields

$$\frac{q^2}{4\pi\epsilon_0 r_1} = \frac{q^2}{4\pi\epsilon_0 r_2} + \frac{1}{2}mv^2.$$

The solution for v is

$$v = \sqrt{\frac{2q^2}{4\pi\epsilon_0 m} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2)(3.1 \times 10^{-6} \text{ C})^2}{20 \times 10^{-6} \text{ kg}} \left(\frac{1}{0.90 \times 10^{-3} \text{ m}} - \frac{1}{2.5 \times 10^{-3} \text{ m}} \right)}$$
$$= 2.5 \times 10^3 \text{ m/s}.$$