3. The total induced emf is given by

$$\varepsilon=-N\frac{d\Phi_B}{dt}=-NA\left(\frac{dB}{dt}\right)=-NA\frac{d}{dt}(\mu_0 ni)=-N\mu_0 nA\frac{di}{dt}=-N\mu_0 n(\pi r^2)\frac{di}{dt}$$

$$=-(120)(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(22000/\mathrm{m})\,\pi\left(0.016\mathrm{m}\right)^2\left(\frac{1.5\ \mathrm{A}}{0.025\ \mathrm{s}}\right)$$

$$=0.16\mathrm{V}.$$

Ohm's law then yields $i=|\varepsilon|/R=0.016\ \mathrm{V}/5.3\Omega=0.030\ \mathrm{A}$.

7. (a) The magnitude of the emf is

$$|\varepsilon|=\left|\frac{d\Phi_B}{dt}\right|=\frac{d}{dt}\left(6.0t^2+7.0t\right)=12t+7.0=12\left(2.0\right)+7.0=31\,\mathrm{mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through *R*.

11. (a) It should be emphasized that the result, given in terms of $\sin(2\pi ft)$, could as easily be given in terms of $\cos(2\pi ft)$ or even $\cos(2\pi ft+\phi)$ where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta+\phi)$. Here our choice is such that $\Phi_B=BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta=\omega t$ (equivalent to $\theta=2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon=-N\frac{d\left(BA\cos\theta\right)}{dt}=-NBA\frac{d\cos\left(2\pi ft\right)}{dt}=N\,Bab2\pi f\sin\left(2\pi ft\right)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0\sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0=2\pi f\,NabB$.

(b) We solve

$$\varepsilon_0=150\ \mathrm{V}=2\pi f\,NabB$$

when $f=60.0$ rev/s and $B=0.500$ T. The three unknowns are *N*, *a*, and *b* which occur in a product; thus, we obtain $Nab=0.796\ \mathrm{m}^2$.

15. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2 B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00 \text{ m})^2}{2}(0.870 \text{ T/s}) = 1.74 \text{ V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0 \text{ V} + 1.74 \text{ V} = 21.7 \text{ V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

21. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40 \text{ rev/s})(2\pi \text{ rad/rev})}{2\pi} = 40 \text{ Hz}.$$

(b) First, we define angle relative to the plane of Fig. 30-44, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0, which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$, then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi f t$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + (\pi a^2/2)\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\frac{d\cos(2\pi f t)}{dt}$$

which yields $\varepsilon = B\pi^2 a^2 f \sin(2\pi f t)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020 \text{ T})\pi^2 (0.020 \text{ m})^2 (40/\text{s}) = 3.2\times 10^{-3}\,\text{V}.$$

34. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R}\ ,$$

which yields $v_t = mgR/B^2L^2$.