

5. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6} \text{ C}}{\sqrt{(1.10 \times 10^{-3} \text{ H})(4.00 \times 10^{-6} \text{ F})}} = 4.52 \times 10^{-2} \text{ A}.$$

11. (a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f . Consequently, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365 \text{ pF}}}{\sqrt{10 \text{ pF}}} = 6.0.$$

(b) An additional capacitance C is chosen so the ratio of the frequencies is

$$r = \frac{1.60 \text{ MHz}}{0.54 \text{ MHz}} = 2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads (pF), then

$$\frac{\sqrt{C + 365 \text{ pF}}}{\sqrt{C + 10 \text{ pF}}} = 2.96.$$

The solution for C is

$$C = \frac{(365 \text{ pF}) - (2.96)^2(10 \text{ pF})}{(2.96)^2 - 1} = 36 \text{ pF}.$$

(c) We solve $f = 1/2\pi\sqrt{LC}$ for L . For the minimum frequency, $C = 365 \text{ pF} + 36 \text{ pF} = 401 \text{ pF}$ and $f = 0.54 \text{ MHz}$. Thus

$$L = \frac{1}{(2\pi)^2 Cf^2} = \frac{1}{(2\pi)^2 (401 \times 10^{-12} \text{ F})(0.54 \times 10^6 \text{ Hz})^2} = 2.2 \times 10^{-4} \text{ H}.$$

13. (a) The charge (as a function of time) is given by $q = Q \sin \omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q \cos \omega t,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC}$,

$$Q = I\sqrt{LC} = (2.00 \text{ A})\sqrt{(3.00 \times 10^{-3} \text{ H})(2.70 \times 10^{-6} \text{ F})} = 1.80 \times 10^{-4} \text{ C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2 \omega \sin \omega t \cos \omega t}{C}$$

We use the trigonometric identity $\cos \omega t \sin \omega t = \frac{1}{2} \sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C} \sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4} \sqrt{LC} = \frac{\pi}{4} \sqrt{(3.00 \times 10^{-3} \text{ H})(2.70 \times 10^{-6} \text{ F})} = 7.07 \times 10^{-5} \text{ s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C) \sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt} \right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00 \times 10^{-3} \text{ H})(2.70 \times 10^{-6} \text{ F})} = 5.655 \times 10^{-4} \text{ s}$, so

$$\left(\frac{dU_E}{dt} \right)_{\max} = \frac{\pi (1.80 \times 10^{-4} \text{ C})^2}{(5.655 \times 10^{-4} \text{ s})(2.70 \times 10^{-6} \text{ F})} = 66.7 \text{ W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

23. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2 L}{2} = \frac{(3.80 \times 10^{-6} \text{ C})^2}{2(7.80 \times 10^{-6} \text{ F})} + \frac{(9.20 \times 10^{-3} \text{ A})^2 (25.0 \times 10^{-3} \text{ H})}{2} = 1.98 \times 10^{-6} \text{ J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2(7.80 \times 10^{-6} \text{ F})(1.98 \times 10^{-6} \text{ J})} = 5.56 \times 10^{-6} \text{ C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.98 \times 10^{-6} \text{ J})}{25.0 \times 10^{-3} \text{ H}}} = 1.26 \times 10^{-2} \text{ A}.$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q \cos \phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80 \times 10^{-6} \text{ C}}{5.56 \times 10^{-6} \text{ C}}\right) = \pm 46.9^\circ.$$

For $\phi = +46.9^\circ$ the charge on the capacitor is decreasing, for $\phi = -46.9^\circ$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$. We obtain $-\omega Q \sin \phi$, which we wish to be positive. Since $\sin(+46.9^\circ)$ is positive and $\sin(-46.9^\circ)$ is negative, the correct value for increasing charge is $\phi = -46.9^\circ$.

26. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{\max}^2/2C$, where q_{\max} is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{\max}^2}{2C} = \frac{1}{2} \frac{Q^2}{2C} \Rightarrow q_{\max} = \frac{Q}{\sqrt{2}}.$$

Now q_{\max} (referred to as the *exponentially decaying amplitude* in Section 31-5) is related to Q (and the other parameters of the circuit) by

$$q_{\max} = Qe^{-Rt/2L} \Rightarrow \ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}.$$

Setting $q_{\max} = Q/\sqrt{2}$, we solve for t :

$$t = -\frac{2L}{R} \ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2L}{R} \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R} \ln 2.$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.