3. (a) We use Gauss' law for magnetism: $\oint \vec{B}\cdot d\vec{A}=0$. Now,

$$\oint \vec{B}\cdot d\vec{A}=\Phi_1+\Phi_2+\Phi_C,$$

where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is Φ_1 = –25.0 μWb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. Its value is

$$\Phi_2 = \pi(0.120\,\text{m})^2(1.60\times10^{-3}\,\text{T}) = +7.24\times10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}\ .$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1-\Phi_2 = 25.0\,\mu\text{Wb}-72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}\ .$$

Thus, the magnitude is $|\Phi_C| = 47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_C indicates that the flux is inward through the curved surface.

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\,(0.60\ \text{V}\cdot\text{m/s})\frac{r}{R}\,.$$

Using r = 0.0200 m (which, in any case, cancels out) and R = 0.0300 m, we obtain

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi R} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0300\ \text{m})}$$
$$= 3.54\times10^{-17}\ \text{T}\,.$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when $r = R$ in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set r = 0.0500 m and solve. We now find

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi r} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0500\ \text{m})}$$
$$= 2.13\times10^{-17}\ \text{T}.$$

9. (a) Application of Eq. 32-7 with $A = \pi r^2$ (and taking the derivative of the field expression given in the problem) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi r^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

For r = 0.0200 m, this gives

$$\begin{aligned} B &= \frac{1}{2}\varepsilon_0\mu_0 r(0.00450\ \text{V/m}\cdot\text{s}) \\ &= \frac{1}{2}(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.0200\ \text{m})(0.00450\ \text{V/m}\cdot\text{s}) \\ &= 5.01\times10^{-22}\ \text{T}. \end{aligned}$$

(b) With $r > R$, the expression above must replaced by

$$B(2\pi r) = \varepsilon_0\mu_0\pi R^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

Substituting r = 0.050 m and R = 0.030 m, we obtain $B = 4.51 \times 10^{-22}$ T.

17. (a) Using Eq. 27-10, we find $E = \rho J = \dfrac{\rho i}{A} = \dfrac{(1.62\times10^{-8}\,\Omega\cdot\text{m})(100\,\text{A})}{5.00\times10^{-6}\,\text{m}^2} = 0.324\ \text{V/m}.$

(b) The displacement current is

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0 A\frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0\rho\frac{di}{dt} = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.62\times10^{-8}\,\Omega\right)\left(2000\,\text{A/s}\right)$$
$$= 2.87\times10^{-16}\,\text{A}.$$

(c) The ratio of fields is $\dfrac{B(\text{due to}\, i_d)}{B(\text{due to}\, i)} = \dfrac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \dfrac{i_d}{i} = \dfrac{2.87\times10^{-16}\ \text{A}}{100\,\text{A}} = 2.87\times10^{-18}.$

23. The electric field between the plates in a parallel-plate capacitor is changing, so there is a nonzero displacement current $i_d = \varepsilon_0(d\Phi_E/dt)$ between the plates.

Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. The current into the positive plate of the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{d(Ed)}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0\frac{d\Phi_E}{dt},$$

which is the same as the displacement current.

(a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_d = i = 2.0$ A.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0\frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\text{A}}{\left(8.85\times10^{-12}\ \text{F/m}\right)\left(1.0\,\text{m}\right)^2} = 2.3\times10^{11}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

(c) The displacement current through the indicated path is

$$i'_d = i_d\left(\frac{d^2}{L^2}\right) = (2.0\,\text{A})\left(\frac{0.50\,\text{m}}{1.0\,\text{m}}\right)^2 = 0.50\,\text{A}.$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i'_d = \left(1.26\times10^{-16}\ \text{H/m}\right)\left(0.50\,\text{A}\right) = 6.3\times10^{-7}\ \text{T}\cdot\text{m}.$$

24. (a) From Eq. 32-10,

$$\begin{aligned} i_d &= \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0 A\frac{dE}{dt}\varepsilon_0 A\frac{d}{dt}\left[\left(4.0\times10^5\right)-\left(6.0\times10^4 t\right)\right] = -\varepsilon_0 A\left(6.0\times10^4\ \text{V/m}\cdot\text{s}\right) \\ &= -\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.0\times10^{-2}\,\text{m}^2\right)\left(6.0\times10^4\ \text{V/m}\cdot\text{s}\right) \\ &= -2.1\times10^{-8}\,\text{A}. \end{aligned}$$

Thus, the magnitude of the displacement current is $|i_d| = 2.1\times10^{-8}\,\text{A}$.

(b) The negative sign in i_d implies that the direction is downward.

(c) If one draws a counterclockwise circular loop s around the plates, then according to Eq. 32-18,

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.