

30. (a) The flux through Arizona is

$$\Phi = -B_r A = -(43 \times 10^{-6} \text{ T})(295,000 \text{ km}^2)(10^3 \text{ m/km})^2 = -1.3 \times 10^7 \text{ Wb} ,$$

inward. By Gauss' law this is equal to the negative value of the flux  $\Phi'$  through the rest of the surface of the Earth. So  $\Phi' = 1.3 \times 10^7 \text{ Wb}$ .

(b) The direction is outward.

31. The horizontal component of the Earth's magnetic field is given by  $B_h = B \cos \phi_i$ , where  $B$  is the magnitude of the field and  $\phi_i$  is the inclination angle. Thus

$$B = \frac{B_h}{\cos \phi_i} = \frac{16 \mu\text{T}}{\cos 73^\circ} = 55 \mu\text{T} .$$

47. (a) If the magnetization of the sphere is saturated, the total dipole moment is  $\mu_{\text{total}} = N\mu$ , where  $N$  is the number of iron atoms in the sphere and  $\mu$  is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with  $N$  iron atoms. The mass of such a sphere is  $Nm$ , where  $m$  is the mass of an iron atom. It is also given by  $4\pi\rho R^3/3$ , where  $\rho$  is the density of iron and  $R$  is the radius of the sphere. Thus  $Nm = 4\pi\rho R^3/3$  and

$$N = \frac{4\pi\rho R^3}{3m} .$$

We substitute this into  $\mu_{\text{total}} = N\mu$  to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3 \mu}{3m} \Rightarrow R = \left( \frac{3m\mu_{\text{total}}}{4\pi\rho\mu} \right)^{1/3} .$$

The mass of an iron atom is  $m = 56 \text{ u} = (56 \text{ u})(1.66 \times 10^{-27} \text{ kg/u}) = 9.30 \times 10^{-26} \text{ kg}$ . Therefore,

$$R = \left[ \frac{3(9.30 \times 10^{-26} \text{ kg})(8.0 \times 10^{22} \text{ J/T})}{4\pi(14 \times 10^3 \text{ kg/m}^3)(2.1 \times 10^{-23} \text{ J/T})} \right]^{1/3} = 1.8 \times 10^5 \text{ m} .$$

(b) The volume of the sphere is  $V_s = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} (1.82 \times 10^5 \text{ m})^3 = 2.53 \times 10^{16} \text{ m}^3$  and the volume of the Earth is

$$V_e = \frac{4\pi}{3} (6.37 \times 10^6 \text{ m})^3 = 1.08 \times 10^{21} \text{ m}^3 ,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{2.53 \times 10^{16} \text{ m}^3}{1.08 \times 10^{21} \text{ m}^3} = 2.3 \times 10^{-5}.$$

52. The Curie temperature for iron is  $770^\circ\text{C}$ . If  $x$  is the depth at which the temperature has this value, then  $10^\circ\text{C} + (30^\circ\text{C}/\text{km})x = 770^\circ\text{C}$ . Therefore,

$$x = \frac{770^\circ\text{C} - 10^\circ\text{C}}{30^\circ\text{C}/\text{km}} = 25 \text{ km}.$$