Final Exam, Phyx 135-2

1) A very long solenoid with 10 turns of wire per cm and a radius of 7.0 cm is carrying a current of 20 mA. In the exact center of the solenoid is a very long straight wire that is carrying a current of 6.0 A.

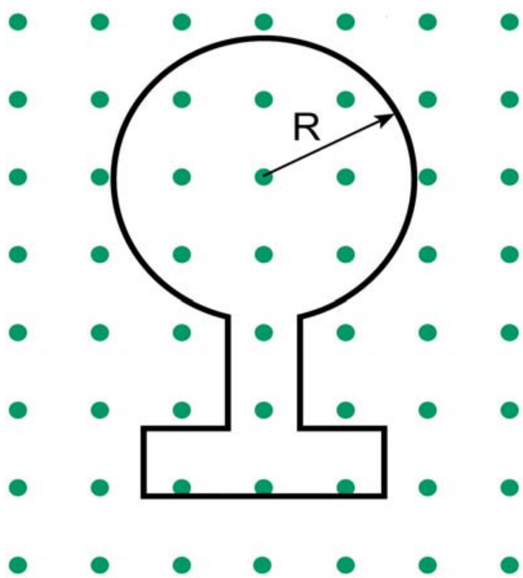
a) (6 points) At what radius from the center will the direction of the net magnetic field be at 45 degrees to the axial direction?
b) (4 points) What is the magnitude of the magnetic field at that radius?

Solution:
a) The direction of the magnetic field due to the solenoid is directly along the axis, whereas the direction of the magnetic field due to the wire (from the right-hand rule) is at 90 degrees to the axial direction. For the two fields to add up to 45 degrees, they must be equal to each other.

So, using $B = \mu_o in$ and $B = \mu_o i/2\pi r$, we have $\mu_o i_S n = \mu_o i_W/2\pi r$, where i_S = current in the solenoid and i_W = current in the wire.

$r = i_W / 2\pi\, i_S\, n = (6\ A) / 2\pi\ (20 \times 10^{-3}\ A)\ (10\ /\ cm) = 4.77\ cm$

b) We could get this answer by calculating the field due to the wire, then calculating the field due to the solenoid, then squaring both and taking the square root. However, since we have already specified that the two magnetic fields are equal, the easy way to get the answer is just to multiply either one by $\sqrt{2}$. We have $\sqrt{2}\ \mu_o i_S n = \sqrt{2}(4\pi \times 10^{-7})(20 \times 10^{-3})(10 / 0.01) = 3.55 \times 10^{-5}$ T



2a) (10 points) A wire is setting in a uniform magnetic field. The normal to the area of the wire is parallel to the magnetic field. The wire is bent into the shape of a circle with a radius of R = 2.3 m, connected to a rectangle of width 1 m and height 2 m, connected to a rectangle of width 3 m and height 0.8 m. (You may ignore any overlap between the circle and the rectangles.) Then the magnetic field begins to change at a rate of 6 T / minute. If the resistance of the wire is 4 Ω, what is the current in the wire?

2b) (5 points) Suppose I hold the magnetic field in Part 2a constant at 2.4 T, and instead begin to shrink only the radius of the circular part of the wire at a rate of 1.5 cm/sec. What would be the current in the wire then (at the instant I began shrinking it)?

Solutions: The emf in the wire will be given by $d\Phi/dt$, where $\Phi = BA\cos\theta$.

2a) In this case we have $d\Phi/dt = (dB/dt)\, A \cos\theta$, because A and θ are constant.

So, dB/dt = 6 / 60 = 0.1 T /sec,

$A = \pi(2.3\text{ m})^2 + (1\text{ m})(2\text{ m}) + (3\text{ m})(0.8\text{ m}) = 21.02\text{ m}^2$

$\cos\theta = 1$

This gives us an emf = 2.102 volts.

V = IR means I = V / R = 2.102 / 4 = 0.526 amp

2b) In this case we have $d\Phi/dt = B\,(dA/dt)\cos\theta$, because B and θ are constant.

So, B = 2.4 T,

$dA/dt = d(\pi r^2)/dt = 2\pi r\, dr/dt = 2\pi(2.3\text{ m})(0.015\text{ m/s}) = 0.2168\text{ m}^2/\text{s}$,

$\cos\theta = 1$

This gives us an emf of (2.4)(0.2168)(1) = 0.52 volts.

V = IR means I = V / R = 0.52 / 4 = 0.13 amp

3a) (5 points) A generator with an adjustable frequency of oscillation is wired in series to an inductor of L = 2.5 mH and a capacitor of C = 3.0 μF. The rms emf of the generator is 110 volts. At what frequency (cycles per second) does the generator produce the largest possible current amplitude in the circuit?

Solution:
The largest possible current will occur at resonance, i.e., at $\omega^2 = 1/LC$. We have:
$\omega = [(2.5 \times 10^{-3})(3 \times 10^{-6})]^{-1/2} = 11{,}547$ rad/sec, or $f = \omega/2\pi = 1838$ Hz.

3b) (5 points) Suppose that the circuit in Problem 3 has a resistance of 25 Ω. What would be the average power dissipated by the circuit at $\omega = 8000$ rad/sec?

Solution:
We have: $V_{rms} = I_{rms}Z$ and $P_{ave} = I_{rms}^2R$, or $P_{ave} = V_{rms}^2 R/Z^2$.
But $Z^2 = R^2 + (X_L - X_C)^2$ where $X_L = \omega L$ and $X_C = 1/\omega C$. Inserting numbers, we have:
$X_L = (8000)(2.5 \times 10^{-3}) = 20\ \Omega$
$X_C = 1/(8000)(3 \times 10^{-6}) = 41.67\ \Omega$
$Z^2 = [25^2 + (41.67 - 20)^2] = 1094.4\ \Omega^2$
$P_{ave} = (110)^2(25)/(1094) = 276$ watts

4) (5 points) I have a laser pointer that produces 3 mW of light. The pointer has a mass of 100 grams. If I accidently left the pointer on and lost it while space-walking (i.e., if I left the laser pointer in a weightless, frictionless environment), how fast would the pointer be moving two hours later?

Solution: The force produced by an electromagnetic wave (light) is $F = P/c$. We have: $F = (3 \times 10^{-3})/(3 \times 10^{8}) = 10^{-11}$ N. This gives us an acceleration of $a = F/m = 10^{-10}$ m/s^2. From $v = at$ we have: $v = (10^{-10} \text{ m/s}^2)(2 \text{ hr})(60 \text{ m/hr})(60 \text{ s/m}) = 7.2 \times 10^{-7}$ m/s.

5) A length of wire 30 meters long is wound into a circular coil with N = 4 turns. The coil is placed into an 8 T magnetic field at an angle of 35 degrees (between the normal to the coil and the field). If a current of 6 mA is running through the coil, then:

a) (5 points) What is the magnetic moment of the coil?

b) (5 points) What torque is acting on the coil?

Solutions:

The magnetic moment μ of a coil is given by $\mu = NiA$. In this case N = 4, $i = 6 \times 10^{-3}$, and the radius of the coil is given by $2\pi r = 30/4 = 7.5$ meters, or r = 1.194 m. This yields $A = \pi(1.194)^2 = 4.476\ m^2$.

a) $\mu = NiA = (4)(6 \times 10^{-3})(4.476) = 0.107$ N-m / T

b) From $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$, we have: $\tau = \mu B \sin\theta = (0.107)(8)\sin(35) = 0.493$ N-m.

6) (10 points) Suppose you have a uniform ball of radius R = 5 cm and charge Q = 0.002 coulomb. (The charge is evenly distributed throughout the sphere.) What would be the electric field inside the ball at a radius of r = 2 cm?

Solution

From Gauss's Law, $\oint \underline{\mathbf{E}} \cdot \underline{\mathbf{dA}} = q_{enc} / \varepsilon_o$. The fact that we have spherical symmetry means that the integral can be rewritten as $E(4\pi r^2)$. The charge enclosed will just be the ratio of the volume at r = 2 cm to the volume at R = 5 cm, or $q_{enc} = Q(^4/_3\pi r^3) / (^4/_3\pi R^3) = Q(r/R)^3$. This gives us:

$E(4\pi r^2) = Q(r/R)^3 / \varepsilon_0$, or $E = Q(^2/_5)^3 / 4\pi\varepsilon_o r^2$

Inserting numbers:

$E = (0.002)(0.064) / (12.56637)(8.85 \times 10^{-12})(0.02)^2 = 2.877 \times 10^9$ V/m

Multiple Choice Questions. (2 points each) Please use capital letters for your answers.

__E__ 6) The magnetic field of the planet Jupiter is:
A) Created by a solid ball of metallic hydrogen.
B) Created by the rotation of the planet.
C) Created by the liquid convection of metallic hydrogen.
D) Created by magnetic monopoles.
E) Both B and C
F) Both A and B

__D__ 7) A current-carrying loop of wire is placed into a uniform magnetic field at an angle of 45 degrees. It has no commutator. The loop will:
A) Spin continuously.
B) Remain still.
C) Flip left or right 90 degrees depending on the direction of the current in the wire.
D) Rotate 45 degrees.
E) Oscillate between 45 degrees and 0 degrees.
F) Rotate 180 degrees.

__A__ 8) If two long, straight current-carrying wires are placed parallel to each other, they will:
A) Attract each other if the currents are in the same direction.
B) Repel each other if the currents are in the same direction.
C) Always attract.
D) Always repel.
E) Attract each other if the currents are in opposite directions.
F) May attract, depending on the current direction, but never repel.

__B__ 9) A transformer can:
A) Only work with DC current.
B) Only work with AC current.
C) Work with either AC or DC current.
D) Step voltages up with DC current.
E) Only work efficiently with AC current at high voltages.
F) Produce negative voltages.

__C__ 10) The so-called "displacement current" is:
A) Not measured in amps.
B) Only produced by RLC circuits.
C) Only created by a changing electric flux.
D) Only created by a changing magnetic flux.
E) Only created by electromagnetic waves.
F) Another name for Faraday induction.