24. (a) Using notation common to many vector-capable calculators, we have (from Eq. 7-8) W = dot([20.0,0] + [0, –(3.00)(9.8)], [0.500 ∠ 30.0º]) = +1.31 J , where “dot” stands for dot product.

(b) Eq. 7-10 (along with Eq. 7-1) then leads to $v = \sqrt{2(1.31 \text{ J})/(3.00 \text{ kg})}$ = 0.935 m/s.

31. **THINK** The applied force varies with x, so an integration is required to calculate the work done on the body.

EXPRESS As the body moves along the x axis from x_i = 3.0 m to x_f = 4.0 m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} -6x \, dx = -3(x_f^2 - x_i^2) = -3\,(4.0^2 - 3.0^2) = -21 \text{ J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). Given v_i, we can readily calculate v_f.

ANALYZE (a) The work-kinetic theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21 \text{ J})}{2.0 \text{ kg}} + (8.0 \text{ m/s})^2} = 6.6 \text{ m/s}.$$

(b) The velocity of the particle is v_f = 5.0 m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$W = \Delta K \quad \Rightarrow \quad -3\left(x_f^2 - x_i^2\right) = \frac{1}{2} m \left(v_f^2 - v_i^2\right).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0 \text{ kg}}{6 \text{ N/m}}\left((5.0 \text{ m/s})^2 - (8.0 \text{ m/s})^2\right) + (3.0 \text{ m})^2} = 4.7 \text{ m}.$$

LEARN Since $x_f > x_i$, $W = -3(x_f^2 - x_i^2) < 0$, i.e., the work done by the force is negative. From the work-kinetic energy theorem, this implies $\Delta K < 0$. Hence, the speed of the particle will continue to decrease as it moves in the +x-direction.

49. **THINK** We have a loaded elevator moving upward at a constant speed. The forces involved are: gravitational force on the elevator, gravitational force on the counterweight, and the force by the motor via cable.

EXPRESS The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W = W_e + W_c + W_m .$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero, i.e., $W = \Delta K = 0$.

ANALYZE The elevator moves *upward* through 54 m, so the work done by gravity on it is

$$W_e = -m_e g d = -(1200 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = -6.35 \times 10^5 \text{ J}.$$

The counterweight moves *downward* the same distance, so the work done by gravity on it is

$$W_c = m_c g d = (950 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = 5.03 \times 10^5 \text{ J}.$$

Since $W = 0$, the work done by the motor on the system is

$$W_m = -W_e - W_c = 6.35 \times 10^5 \text{ J} - 5.03 \times 10^5 \text{ J} = 1.32 \times 10^5 \text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0 \text{ min} = 180 \text{ s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_m}{\Delta t} = \frac{1.32 \times 10^5 \text{ J}}{180 \text{ s}} = 7.4 \times 10^2 \text{ W}.$$

LEARN In general, the work done by the motor is $W_m = (m_e - m_c) g d$. So when the counterweight mass balances the total mass, $m_c = m_e$, no work is required by the motor.

62. (a) The compression of the spring is d = 0.12 m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25 \text{ kg}) \left(9.8 \text{ m/s}^2\right) (0.12 \text{ m}) = 0.29 \text{ J}.$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2} k d^2 = -\frac{1}{2} \; (250 \text{ N/m}) \; (0.12 \text{ m})^2 = -1.8 \text{ J}.$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15):

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29\text{ J} - 1.8\text{ J})}{0.25\text{ kg}}} = 3.5\text{ m/s}.$$

(d) If we instead had $v_i' = 7\,\text{m/s}$, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$0 - \frac{1}{2}mv_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2}kd'^2$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^2}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this result, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048}\text{ m/s} = 3.471\text{ m/s}$ and v_i' = 6.942 m/s).

68. Using Eq. 7-7, we have $W = Fd \cos\phi = 1504\text{ J}$. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.