20. (a) We take the reference point for gravitational energy to be at the lowest point of the swing. Let θ be the angle measured from vertical. Then the height y of the pendulum "bob" (the object at the end of the pendulum, which in this problem is the stone) is given by $L(1 - \cos\theta) = y$. Hence, the gravitational potential energy is

$$mg\,y = mgL(1 - \cos\theta).$$

When $\theta = 0°$ (the string at its lowest point) we are told that its speed is 8.0 m/s; its kinetic energy there is therefore 64 J (using Eq. 7-1). At $\theta = 60°$ its mechanical energy is

$$E_{\text{mech}} = \frac{1}{2}\,mv^2 + mgL(1 - \cos\theta)\,.$$

Energy conservation (since there is no friction) requires that this be equal to 64 J. Solving for the speed, we find v = 5.0 m/s.

(b) We now set the above expression again equal to 64 J (with θ being the unknown) but with zero speed (which gives the condition for the maximum point, or "turning point" that it reaches). This leads to $\theta_{\max} = 79°$.

(c) As observed in our solution to part (a), the total mechanical energy is 64 J.

30. We take the original height of the box to be the y = 0 reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d\sin 40°$.

(a) Using the conservation of energy, we have

$$K_i + U_i = K + U \Rightarrow 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2.$$

Therefore, with d = 0.10 m, we obtain v = 0.81 m/s.

(b) We look for a value of $d \neq 0$ such that K = 0.

$$K_i + U_i = K + U \Rightarrow 0 + 0 = 0 + mgy + \frac{1}{2}kd^2\,.$$

Thus, we obtain $mgd\,\sin 40° = \frac{1}{2}kd^2$ and find d = 0.21 m.

(c) The uphill force is caused by the spring (Hooke's law) and has magnitude kd = 25.2 N. The downhill force is the component of gravity $mg\sin 40°$ = 12.6 N. Thus, the net force on the box is (25.2 – 12.6) N = 12.6 N uphill, with

$$a = F/m = (12.6\ \text{N})/(2.0\ \text{kg}) = 6.3\ \text{m/s}^2.$$

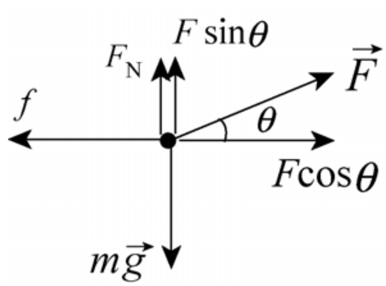
(d) The acceleration is up the incline.

45. **THINK** Work is done against friction while pulling a block along the floor at a constant speed.

EXPRESS Place the x-axis along the path of the block and the y-axis normal to the floor. The free-body diagram is shown below. The x and the y component of Newton's second law are

$$x: \quad F\cos\theta - f = 0$$
$$y: \quad F_N + F\sin\theta - mg = 0,$$

where m is the mass of the block, F is the force exerted by the rope, f is the magnitude of the kinetic friction force, and θ is the angle between that force and the horizontal.



The work done on the block by the force in the rope is $W = Fd\cos\theta$. Similarly, the increase in thermal energy of the block-floor system due to the frictional force is given by Eq. 8-29, $\Delta E_{\text{th}} = fd$.

ANALYZE (a) Substituting the values given, we find the work done on the block by the rope's force to be

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1\ \text{J}.$$

(b) The increase in thermal energy is $\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\ \text{J}.$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. The x-component of Newton's law gives

$$f = F\cos\theta = (7.68\ \text{N})\cos 15.0° = 7.42\ \text{N}.$$

Similarly, the y-component yields

$$F_N = mg - F\sin\theta = (3.57\ \text{kg})(9.8\ \text{m/s}^2) - (7.68\ \text{N})\sin 15.0° = 33.0\ \text{N}.$$

Thus, the coefficient of kinetic friction is

$$\mu_k = \frac{f}{F_N} = \frac{7.42\ \text{N}}{33.0\ \text{N}} = 0.225.$$

LEARN In this problem, the block moves at a constant speed so that $\Delta K = 0$, i.e., no change in kinetic energy. The work done by the external force is converted into thermal energy of the system, $W = \Delta E_{\text{th}}$.

57. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where $h = 1.1$ m. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy $\Delta E_{\text{th}} = f_k d = \mu mgd$, where $\mu = 0.60$. Thus, Eq. 8-33 (with $W = 0$) provides us with an equation to solve for the distance d:

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\left(h + \mu d\right)$$

where $K_i = mv_i^2/2$ and $v_i = 6.0$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2\ \text{m}.$$

58. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 40°$), which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$ where $\mu_k = 0.15$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = mgd\sin\theta$. Eq. 8-33 (with $W = 0$ and $K_f = 0$) provides an equation for determining d:

$$K_i = \Delta U + \Delta E_{\text{th}}$$

$$\frac{1}{2}mv_i^2 = mgd\left(\sin\theta + \mu_k\cos\theta\right)$$

where $v_i = 1.4\ \text{m/s}$. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)} = 0.13\ \text{m}.$$

(b) Now that we know where on the incline it stops (d' = 0.13 + 0.55 = 0.68 m from the bottom), we can use Eq. 8-33 again (with W = 0 and now with K_i = 0) to describe the final kinetic energy (at the bottom):

$$K_f = -\Delta U - \Delta E_{\text{th}}$$
$$\frac{1}{2}mv^2 = mgd'(\sin\theta - \mu_k \cos\theta)$$

which — after dividing by the mass and rearranging — yields

$$v = \sqrt{2gd'(\sin\theta - \mu_k \cos\theta)} = 2.7 \text{ m/s}.$$

(c) In part (a) it is clear that d increases if μ_k decreases — both mathematically (since it is a positive term in the denominator) and intuitively (less friction — less energy "lost"). In part (b), there are two terms in the expression for v that imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin\theta - \mu_k \cos\theta$, which indicates that less is being subtracted from $\sin\theta$ when μ_k is less (so the factor itself increases in value).