76. We use Eq. 9-88. Then

$$v_f = v_i + v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) = 105 \text{ m/s} + (253 \text{ m/s}) \ln\left(\frac{6090 \text{ kg}}{6010 \text{ kg}}\right) = 108 \text{ m/s}.$$

77. **THINK** The mass of the faster barge is increasing at a constant rate. Additional force must be provided in order to maintain a constant speed.

EXPRESS We consider what must happen to the coal that lands on the faster barge during a time interval Δt. In that time, a total of Δm of coal must experience a change of velocity (from slow to fast) $\Delta v = v_{\text{fast}} - v_{\text{slow}}$, where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta p}{\Delta t} = \frac{(\Delta m)}{\Delta t}\Delta v = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}})$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$. Note that we ignore the transverse speed of the coal as it is shoveled from the slower barge to the faster one.

ANALYZE (a) With $v_{\text{fast}} = 20 \text{ km/h} = 5.56 \text{ m/s}$, $v_{\text{slow}} = 10 \text{ km/h} = 2.78 \text{ m/s}$ and the rate of mass change $(\Delta m / \Delta t) = 1000 \text{ kg/min} = (16.67 \text{ kg/s})$, the force that must be applied to the faster barge is

$$F_{\text{fast}} = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}}) = (16.67 \text{ kg/s})(5.56 \text{ m/s} - 2.78 \text{ m/s}) = 46.3 \text{ N}$$

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

LEARN The force that must be applied to the faster barge in order to maintain a constant speed is equal to the rate of change of momentum of the coal.

78. We use Eq. 9-88 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\frac{M_i}{M_f} \Rightarrow \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$ we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$ we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.

79. **THINK** As fuel is consumed, both the mass and the speed of the rocket will change.

EXPRESS The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. On the other hand, the mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where Δt is the time interval of the burn. Thus, the mass of the rocket after the burn is

$$M_f = M_i - M_{\text{fuel}} .$$

ANALYZE (a) Given that $R = 480$ kg/s and $v_{\text{rel}} = 3.27 \times 10^3$ m/s, we find the thrust to be

$$T = Rv_{\text{rel}} = (480\,\text{kg/s})(3.27\times10^3\ \text{m/s}) = 1.57\times10^6\ \text{N}.$$

(b) With the mass of fuel ejected given by $M_{\text{fuel}} = R\Delta t =$ (480 kg/s)(250 s) = 1.20×10^5 kg, the final mass of the rocket is

$$M_f = M_i - M_{\text{fuel}} = (2.55 \times 10^5\ \text{kg}) - (1.20 \times 10^5\ \text{kg}) = 1.35 \times10^5\ \text{kg}.$$

(c) Since the initial speed is zero, the final speed of the rocket is

$$v_f = v_{\text{rel}} \ln\frac{M_i}{M_f} = (3.27\times10^3\ \text{m/s})\ln\left(\frac{2.55\times10^5\ \text{kg}}{1.35\times10^5\ \text{kg}}\right) = 2.08\times10^3\ \text{m/s}.$$

LEARN The speed of the rocket continues to rise as the fuel is consumed. From the first rocket equation given in Eq. 9-87, the thrust of the rocket is related to the acceleration by $T = Ma$. Using this equation, we find the initial acceleration to be

$$a_i = \frac{T}{M_i} = \frac{1.57\times10^6\ \text{N}}{2.55\times10^5\ \text{kg}} = 6.16\ \text{m/s}^2.$$