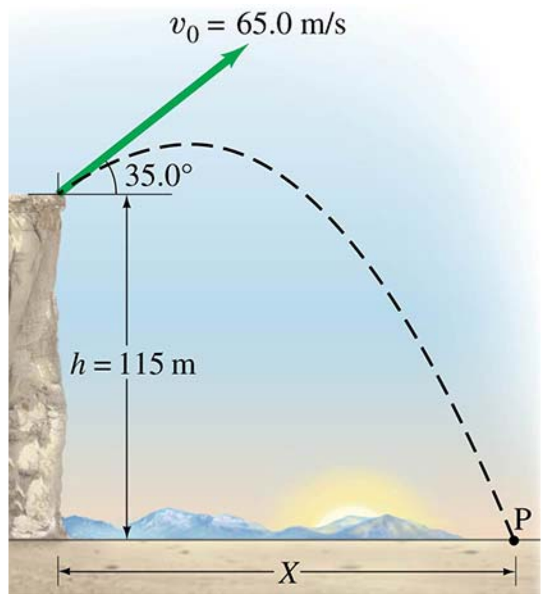
Phyx 135-1 Midterm 1, Spring 18 **Solution Key**

1) (10 points) You are at the top of a cliff which is 115 meters high, as shown at right. Then you throw a baseball off the cliff at 65 m/s and an angle of 35°. How far away from the base of the cliff will the baseball hit the ground?



Solution

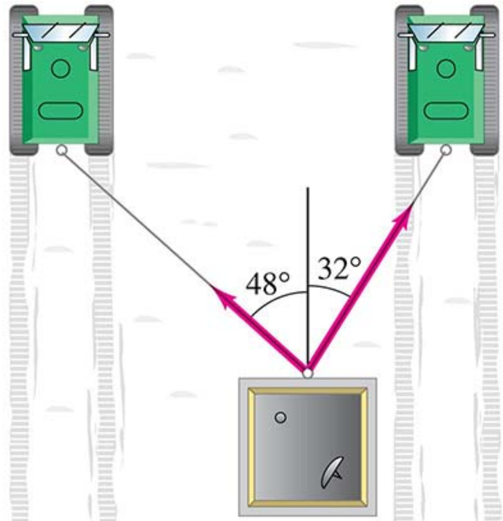
We see that the baseball will start with vector velocity components of: $v_x = 65 \cos(35°) =$ 53.2 m/s, and $v_y = 65 \sin(35°) = 37.3$ m/s.

Next, we can use the basic equation for motion at constant acceleration to find the time that the baseball will be in the air. If set y = 0 at the bottom of the cliff, then we have:

$y = y_0 + v_y t - \frac{1}{2} g t^2$, or $0 = 115 + (37.3)t - \frac{1}{2}(9.8)t^2$. If you had a calculator that would automatically solve the quadratic equation, that was OK, but doing it by hand isn't too hard. We have: $2at = -b \pm (b^2 - 4ac)^{1/2}$, where a = –4.9, b = 37.3 and c = 115. This gives us: $-9.8t = -37.3 \pm (1391.3 + 2254)^{1/2}$, or t = 10 s until the baseball hits the ground. (There is also a solution with a negative time, but that would correspond to the baseball hitting the ground before you threw it, which we reject.)

Finally, we have $x = v_x t = (53.2)(10) =$ 532 meters.

2) (10 points) The figure at right shows a top view of two tractors dragging a heavy sled. The sled has a mass of 1100 kg. If the sled is accelerating due north at 1.2 m/s^2, and it has a coefficient of kinetic friction of 0.15, what are the separate tensions in the left and right cords?



Solution

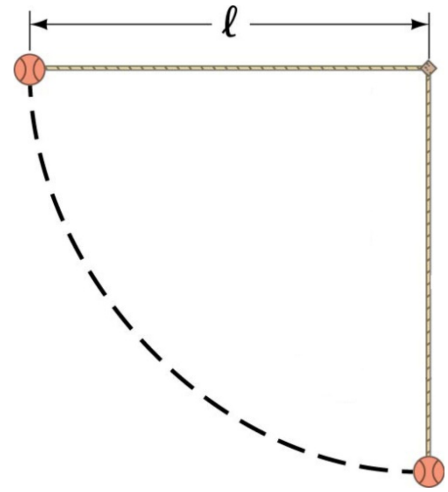
There are two unknown tensions here, so we will need two equations. Since the sled is only moving along the y-axis, we see that the x-components of the tensions in the cords must be opposite and equal. We have: $T_L \sin(48°) = T_R \sin(32°)$. In the y-direction, we have $F = ma + \mu N$, or $T_L \cos(48°) + T_R \cos(32°) = (1100)(1.2) + (0.15)(1100)(9.8)$.

There are several ways to do the algebra. I will solve for T_R from the first equation, then substitute that into the second equation. We have: $T_R = T_L (0.74314)/(0.52992) = 1.402\ T_L$.

$T_L(0.6691) + (1.402)T_L(0.8480) = 1320 + 1617$, or T_L = 1581 N.

We have $T_R = (1.402)(1581)$ = 2216 N.

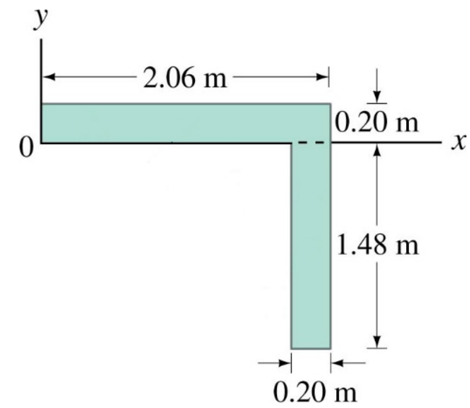
3) (8 points) A ball of mass m = 0.2 kg is attached to a string with a length of l = 30 cm. It is held with the string horizontal, then released. When the string is vertical, how much tension will there be on it?



Solution

At the bottom of the arc, we can use conservation of energy to calculate the ball's velocity. We have $\frac{1}{2} mv^2 = mgh$, or $v^2 = 2gl$. The centrifugal force acting on the ball will therefore be $F = mv^2/l = m(2gl)/l = 2mg$. In addition to this, gravity is operating on the ball and will add another mg to the downward force. Thus T = 3mg = 3(0.2)(9.8) = 5.88 N.

4) (10 points) If the x-y coordinate origin of the object at right is at the large zero, what are the coordinates of the center of mass of the object? You may assume that the object has a uniform thickness and composition.



Solution

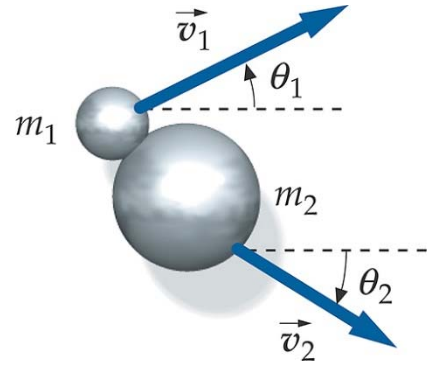
The easiest way to solve this problem is to recall that the CM of an object is the same as the CM of the CM's of its components. If we take the top rectangle to be one component, then it has a CM at (1.03, 0.10), and a "mass" of (2.06)(0.2) = 0.412 units. The lower rectangle has its CM at (2.06 – 0.10, –0.74), or (1.96, –0.74). It has a "mass" of (1.48)(0.2) = 0.296 units.

Using the definition of center of mass gives us:

$x_{cm} = [(0.412)(1.03) + (0.296)(1.96)] / (0.412 + 0.296) = 1.42$

$y_{cm} = [(0.412)(0.1) + (0.296)(-0.74)] / (0.412 + 0.296) = -0.25$

5a) (10 points) A ball with mass m_1 = 1.5 kg is moving at a velocity of v_o = 25 m/s along the positive x-axis. Then it collides with a stationary ball with a mass of m_2 = 6 kg. Afterwards, the small ball ricochets away at an exit angle of θ_1 = 30° and a velocity of v_1 = 16 m/s, as shown at right. What is the final speed and exit angle θ_2 of the large ball?



Solution

We need to know both the x-velocity and the y-velocity of the large ball to answer this question. For the y-velocity, we note that the system initially has no net y-momentum. Therefore, to conserve momentum, $m_1v_1 \sin\theta_1 = m_2v_2 \sin\theta_2$. Inserting numbers: $(1.5)(16)\sin(30°) = (6)v_2 \sin\theta_2$, or $2 = v_2 \sin\theta_2$.

Likewise, to conserve x-momentum, we must have: $m_1v_o = m_1v_1 \cos\theta_1 + m_2v_2 \cos\theta_2$. Inserting numbers: $(1.5)(25) = (1.5)(16) \cos(30°) + (6)v_2 \cos\theta_2$, or $2.786 = v_2 \cos\theta_2$.

If we divide the first equation by the second, then we obtain: $2 / 2.786 = \tan\theta_2$, or $\theta_2 = 35.7°$
Substituting back into the first equation yields: $2 = v_2 \sin(35.7°)$, or $v_2 = 3.43$ m/s.

5b) (2 points) Was this collision elastic? You must prove it one way or the other; guesses will not be counted.

Solution

The small ball initially has a kinetic energy of $E_1 = ½ mv^2 = ½ (1.5)(25)^2 = 469$ J.
Afterwards, the two balls together have $E_2 = ½ (1.5)(16)^2 + ½ (6)(3.43)^2 = 227$ J.

No, the kinetic energies are nowhere near equal, so the collision was inelastic.