Phyx 135-1 Midterm 2, Spring 18 Solution Key

1) A sphere with a mass of M = 5 kg and a radius of R = 25 cm is fixed to a wall by a zero-friction axis through its center. A cord wrapped around its equator is attached to a smaller mass of m = 3 kg.
1a) (5 points) At what acceleration is the smaller mass falling?
1b) (5 points) What is the tension in the cord?

Solution
The torque on the sphere is given by two equations: $\tau = r \times F = -RT$, where T is the tension in the cord, and $\tau = I\alpha = {}^2/_5\, MR^2\, \alpha$. We also know that $a_T = R\alpha$, so we have: $-RT = {}^2/_5\, MR^2\, (a_T/R)$, or $-T = {}^2/_5\, Ma$.

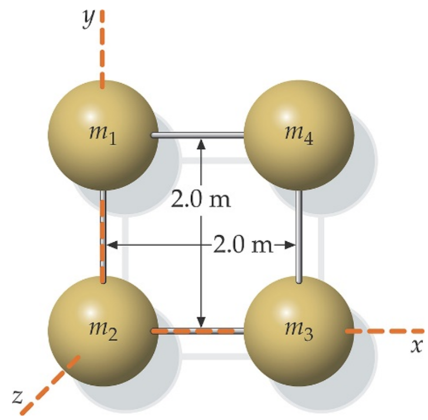
For the small mass, setting up a free-body diagram immediately gives us $T - mg = ma$. If we add this equation to the first one, we get: $-mg = {}^2/_5\, Ma + ma$, or $a = -mg / ({}^2/_5\, M + m) = -(3)(9.8) / (2 + 3) =$ $-5.88\ m/s^2$. Then $T = -{}^2/_5\, Ma = (0.4)(5)(5.88) =$ 11.76 N.

1c) (5 points) If the small mass falls a total distance of 12 m, at what angular speed ω will the sphere be rotating?

Solution
There are two ways to solve this problem. One way is to realize that $d = \frac{1}{2} at^2$. So, for the small mass, $t = (2d/a)^{1/2} = (24/5.88)^{1/2} = 2.02$ s. Then, $\omega = \alpha t = (a/R)t = (5.88/0.25)(2.02) =$ 47.5 rad/s.

The other, longer way is to use conservation of energy. We have $mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$. However, $v_T = R\omega$, so there is really only one variable on the right-hand side. Substituting, $mgh = \frac{1}{2} mR^2\omega^2 + \frac{1}{2}({}^2/_5\, MR^2)\omega^2 = [(1.5)(0.25)^2 + (1)(0.25)^2]\omega^2 = 0.15625\omega^2$. Then $\omega = [(3)(9.8)(12)/0.15625]^{1/2} =$ 47.5 rad/s.

2) (5 points) Suppose you have four identical spheres with m = 2 kg and R = 50 cm, connected by rigid, massless rods. Their centers are set in a square as shown at right. What is the moment of inertia of this assembly around the z axis?



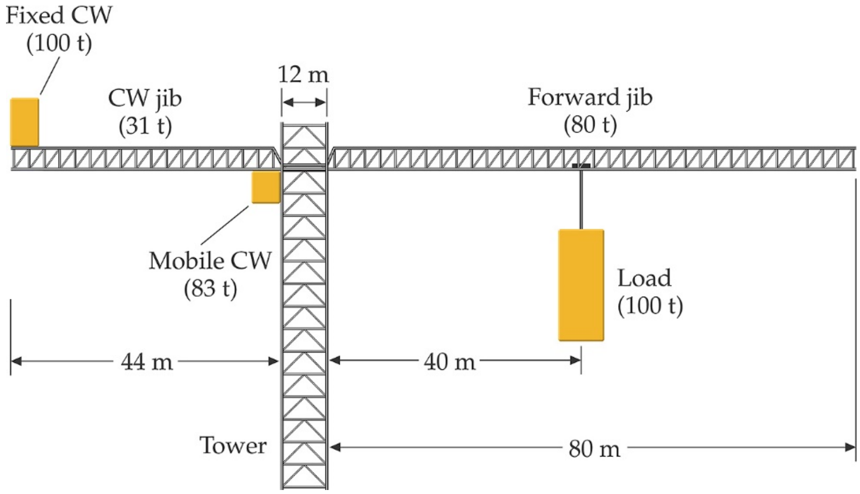
Solution

The moment of inertia of each sphere around its CM is $I = {}^2/_5\, mR^2 = (0.4)(2)(0.5)^2 = 0.2$ kg m^2. To find the moment of inertia of the entire system, we must use the parallel axis theorem. We have $I_{axis} = md^2 + I_{CM}$ for each sphere.

Writing everything out:

$I = (0.2) + 2[(2)(2^2) + 0.2] + [(2)(2^2 + 2^2) + 0.2] =$ 32.8 kg m^2.

3) (8 points) In the illustration at right, a lifting crane has a forward arm that is 80 m long with a mass of 80 metric tons. Its rear arm is 44 m long with a mass of 31 tons. A fixed counter-weight of 100 tons is at the end of the rear arm. If a load of 100 tons is placed at 40 m from the right edge of the central tower, how far from the center of the tower must the 83-ton mobile counter-weight be placed so that there is no net torque around the center of the tower?



Solution

The two arms will act as though their masses are concentrated at their centers. So, the clockwise torque around the center of the tower is given by $\tau = r \times F = Rmg = (40 + 6)(80 + 100)g = 8280g$. The counter-clockwise torque is given by $\tau = (22 + 6)(31)g + (44 + 6)(100)g + 83dg$, where d is the distance of the mobile counter-weight from the center of the tower. Equating the two torques gives us: $8280 = 868 + 5000 + 83d$, or d = 29.06 m.

4) (12 points) A uniform bar of length 80 cm and mass 250 g is hanging from a pivot at its end. Then, a clay bullet with a mass of 60 g and travelling at 400 cm/s strikes the bar perpendicularly at a point 40 cm down from the pivot, and sticks to the bar. What is the maximum angle (as measured from the vertical) to which the bar will rise?

Solution

We cannot start by using conservation of energy, because the collision is inelastic and therefore an unknown amount of the impact will be converted into heat. We must start instead with conservation of angular momentum. The initial angular momentum is given by the momentum of the bullet around the bar's pivot, so we have $L = r \times p = (0.4)(0.06)(4) = 0.096$ kg m^2/s.

The initial ω of the bar-bullet combination can then be found from $L = I\omega$. For I, we have $^1/_3\, ML^2 + md^2 = {}^1/_3(0.25)(0.8)^2 + (0.06)(0.4)^2 = 0.06293$ kg m^2. This yields $\omega = 0.096 / 0.06293 = 1.526$ rad/s. Now then, at this point we *can* use conservation of energy, because this ω is valid regardless of how much heat was generated. The kinetic energy of the bar-bullet combination is $E = ½\, I\omega^2 = (0.5)(0.06293)(1.526)^2 = 0.07327$ J. The center of mass of the bar-bullet will rise to $E = mgh$, or $0.07327 = (0.25 + 0.06)(9.8)h$, or $h = 0.0241$ m $= 2.41$ cm.

We then have $\cos\theta = (40 – 2.41)/40$, or $\theta = 20°$

5) (10 points) You are at a bowling alley. You release a strike which has a total kinetic energy of 90 joules. If the bowling alley is 18.3 meters long, and the ball rolled without slipping, how long did it take before the ball hit the pins? (Assume that the bowling ball has a mass of 6.4 kg, and a radius of 10.8 cm.)

Solution

The rolling ball will have a kinetic energy given by $E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$. However, for a rolling ball $v = R\omega$, so $E = \frac{1}{2} mv^2 + \frac{1}{2} (\frac{2}{5} mR^2)(v/R)^2 = \frac{1}{2} mv^2 + \frac{1}{5} mv^2 = 0.7\ mv^2$. Inserting numbers, $v = [90 / (0.7)(6.4)]^{1/2} = 4.48$ m/s. Then we have $t = d/v = 18.3/4.48 =$ 4.08 s.