

1) (10 points) The interstellar garbage skow Superjunk is floating still in space. It is fully loaded and has a total mass of 70,000 kg. Then it starts up its garbage ejector, which is just a conveyor belt with scoops on it. The belt ejects one scoop of garbage every two seconds at an exit velocity of 210 km/hr, and each scoop contains 100 kg of garbage. The belt runs for two minutes. Then it breaks down. There is a half-hour delay for repairs. The belt runs for another five minutes. Then it breaks down again. There is a 20-minute delay for repairs. Finally, the belt runs for four minutes and finishes its job. How fast is the skow Superjunk moving at this point?

Solution

If we think of the skow as a rocket, and consider the ejected garbage to be “fuel”, then all we have here is a simple rocket. The time delays for repairing the belt are irrelevant. All that matters is the total mass ejected. The belt ran for 2 min + 5 min + 4 min = 11 min. It can deliver 30 scoops of garbage per minute at 100 kg per scoop, or 3000 kg/min. This gives us a total ejected mass of (11)(3000) = 33,000 kg. So, the final mass of Superjunk is 70,000 – 33,000 = 37,000 kg. We then use $v_F = v_0 \ln(M_i / M_f) = (210 \text{ km/hr}) \ln(70,000/37,000) = 134 \text{ km/hr}$.

2) (10 points) A bicycle is hanging by one of its wheels from a nail on a wall in a garage. The bicycle happens to have an *extremely* light frame, so we can think of it as just one wheel (hanging from the nail) plus a second wheel whose center is one meter directly below the center of the first wheel. Both wheels are 62 cm in diameter, and have a mass of 1.5 kg. What is the moment of inertia of the bicycle about the nail? (You may assume that the wheels are simple hoops.)

Solution

We will need to apply the parallel axis theorem twice, once for each wheel. We know that $I_{\text{axis}} = md^2 + I_{\text{cm}}$. We know (or deduce) that I_{cm} for a hoop is just mR^2 , so in our case we have $I_{\text{cm}} = (1.5)(0.31)^2 = 0.144 \text{ kg m}^2$.

For the first wheel, d is the distance from the nail at the edge of the wheel to the center of the wheel, or 0.31 m. Thus $I_{\text{axis}} = (1.5)(0.31)^2 + 0.144 = 0.288 \text{ kg m}^2$.

For the second wheel, $d = 0.31 + 1.0 = 1.31 \text{ m}$. So $I_{\text{axis}} = (1.5)(1.31)^2 + 0.144 = 2.718 \text{ kg m}^2$.

The total moment of inertia is $2.718 + 0.288 = 3.01 \text{ kg m}^2$.