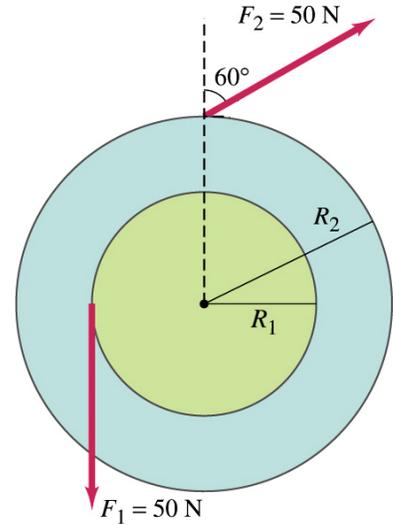


1) (10 points) Two cylinders are welded to each other as shown at right. The smaller cylinder has $R_1 = 2$ m and $m_1 = 3$ kg. The larger cylinder has $R_2 = 3$ m and $m_2 = 5$ kg. Forces are pulling at the rims of each cylinder as shown. What is the angular acceleration of the cylinders? Include the direction, either cc or cw.



Solution

We can use $\tau = r \times F = I\alpha$. Using the standard convention that ccw rotation is positive, we have $\tau = (2 \text{ m})(50 \text{ N}) - (3 \text{ m})(50 \text{ N}) \sin(60^\circ)$, or $\tau = -29.9 \text{ N m}$. The negative sign means that the rotation of the two cylinders is **clockwise**.

The moment of inertia of a cylinder is $\frac{1}{2} MR^2$, so in this case we have $I = \frac{1}{2}(3)2^2 + \frac{1}{2}(5)3^2 = 28.5 \text{ kg m}^2$. Then $\alpha = -29.9 / 28.5 = -1.049 \text{ rad/s}^2$.

2) (10 points) An old-fashioned LP vinyl record is spinning on a turntable at $33 \frac{1}{3}$ rpm. The record has a diameter of 12 inches and a mass of 4.23 oz. The turntable is turned off, and the record spins frictionlessly. However, just as it is turned off, an old 45 rpm single accidentally drops onto the LP and grinds against it until both are rotating together. If the 45 rpm single has a diameter of 7 inches and a mass of 1.41 oz, at how many rpm's will the two records be spinning?

Solution

From conservation of angular momentum, we know that $I_1\omega_1 = I_2\omega_2$. (The parameters for the records are given in English units rather than metric, but that is OK because the units will just cancel out since we have $I\omega$ on both sides of the equation.) The initial angular momentum is $\frac{1}{2} MR^2 \omega = \frac{1}{2} (4.23)(6^2)(33.33) = 2537.75 \text{ oz in}^2 / \text{min}$. The new moment of inertia after the 45 drops is $I = \frac{1}{2} (4.23)(6^2) + \frac{1}{2} (1.41)(3.5^2) = 84.78 \text{ oz in}^2$. The new ω is $2537.75/84.78 = 29.9 \text{ rpm}$.