Physics 135-1, Quiz 6, Spring 2018 **Solution Key**

1) (10 points) In many a spine-tingling war/espionage movie, the intrepid good guy escapes from the bad guys by diving deep under water and hiding. While underwater, he breathes through a small, hollow reed (poking up to the surface) which is always conveniently found growing in pools of water in war/espionage movies.

Assume that the good guy has lungs that are 30 cm across and 30 cm tall. Assume that he not capable of breathing if a 150-lb spy is standing on his chest. (For this problem, you may assume that his lungs have only a front and a back.) How deep underwater can the good guy dive and still breathe? You may assume that the good guy is lying down, so his lungs are at one depth.
(1 lb = 0.454 kg. 1 atm = 1.01×10^5 N/m^2)

Solution
From $P = P_0 + \rho gh$, we see that the difference in pressure between the air at the top of the reed and the air in the good guy's lungs must be $P = F/A = \rho gh$. The maximum force that he can withstand and still breathe is $F = (150)(0.454)g$, and the area of his lungs is $2(0.3)(0.3) = 0.18$ m^2, so the maximum pressure he can withstand is $(150)(0.454)g/0.18 = 378g$ Pa.

Water has a density of 1 g/cc = 1000 kg/m^3, so $378g = 10^3gh$, or h = 0.378 m = 15 inches.

2) (10 points) The Earth's elliptical orbit brings it as close to the Sun as 1.4709×10^{11} m and as far from the Sun as 1.5210×10^{11} m. The mass of the Earth is 5.9736×10^{24} kg, the mass of the Sun is 1.9891×10^{30} kg, and $G = 6.674 \times 10^{-11}$ N m^2 / kg^2. If the Earth's maximum orbital velocity is 30.29 km/s, what is its minimum orbital velocity?

Solution
This is a smoke-and-mirrors problem. The minimum and maximum distances of the Earth's orbit must be directly opposite each other, so therefore they correspond to the maximum and minimum velocities, respectively. We can use conservation of angular momentum to relate one to the other, or $mv_{max}r_{min} = mv_{min}r_{max}$, or $v_{min} = (30.29)(1.4709)/1.521 =$ 29.29 km/s.

Note – The original version of this problem had the wrong minimum/maximum distances for the Earth's orbit, because I forget to convert km to m. I didn't notice because I didn't think that anyone would actually try to use them!

However, I was wrong. Several students did. To do it that way, you needed to remember that the total energy for an elliptical orbit is given by $T = -GmM / 2a = U + K$, where U and K are the usual gravitational and kinetic energies. We have $-GmM / 2a = -GmM / r + \frac{1}{2} mv^2$, so
$v^2 = GM(2/r - 1/a)$. We have $r = 1.521 \times 10^{11}$ m for the point of the minimum orbital velocity, and
$a = [(1.4709 \times 10^{11}) + (1.5210 \times 10^{11})]/2 = 1.49595 \times 10^{11}$ m. Inserting numbers,
$v^2 = (6.674 \times 10^{-11})(1.9891 \times 10^{30})[2/(1.521 \times 10^{11}) - (1/1.49595 \times 10^{11})] = 8.582 \times 10^8$, or
v = 29.29 km/s.