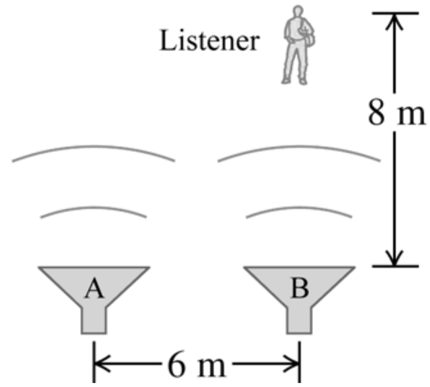
Midterm I, Phyx 135-3, Spring 2018 **Solution Key**

1) Two identical speakers are connected by sophisticated electronics that allows them to produce synchronized sound waves with precisely identical amplitudes, frequencies, and phases. You may assume that sound coming from the speakers radiates uniformly in all directions.



1a) (6 points) When speaker B is turned on but speaker A is turned off, a listener 8 meters from speaker B hears a sound of 75 dB. What sound level in dB does this same listener hear when speaker B is turned off and speaker A is turned on, if the speakers are 6 m apart as shown?

Solution

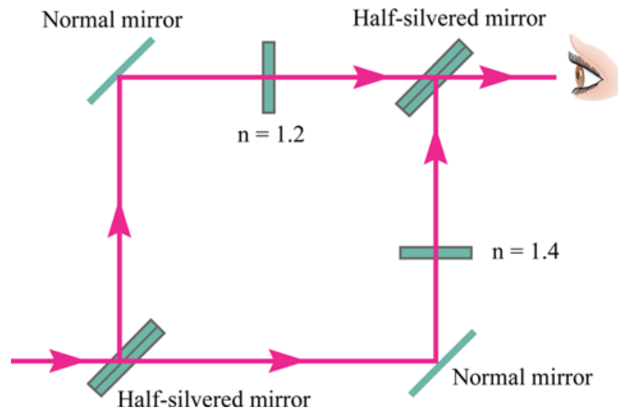
The intensity of a spherical wave decreases as $1/r^2$. Speaker A is $r = (6^2 + 8^2)^{1/2} = 10$ m from the listener, so the sound intensity from speaker A is $8^2 / 10^2 = 0.64$ that of speaker B. To convert dB into intensity, we use $dB = 10 \log (I / I_o)$, so $I = (10^{-12})(10^{75/10}) = 3.16 \times 10^{-5}$ W/m^2 for speaker B. The decibel level from speaker A is $dB = 10 \log [(0.64)(3.16 \times 10^{-5})(10^{12})] = 73.06$

1b) (4 points) If the speakers are emitting an annoying sine wave with a frequency of 1000 Hz, what is the phase angle between the waves from speakers A and B (in degrees) when they reach the listener? You may assume that the speed of sound in air is 343 m/s.

Solution

We have $\delta = 2\pi\, \Delta x / \lambda$, or $\delta = 360° \,\Delta x / \lambda$. From Part A we know that $x_A = 10$ m and $x_B = 8$ m, so $\Delta x = 2$ m. With $v = 343$ m/s, we have $\lambda = v/f = 343/1000 = 0.343$ m. This gives us a phase angle of:
$\delta = 360°(2 \text{ m})/(0.343 \text{ m}) = 2099°$, or 299° after truncating to one cycle.

2) (10 points) Light is sent through the arrangement at right, where it is split into equal parts and moves equal distances before being recombined for viewing. All aspects of the reflections, etc, are identical. There is only one difference. Each beam passes through a sample of equal thickness, but with a different index of refraction: 1.2 and 1.4 respectively. If the observer can see no light at all when the source has $\lambda = 650$ nm, how thick are the samples?



Solution

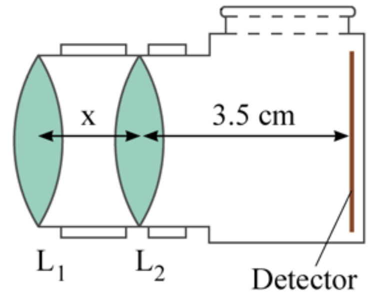
In general, phase shifts are given by $\delta = 2\pi\, \Delta x/\lambda$. In this case the *only* difference between the two waves is the refractive media they move through, where $\lambda = \lambda_{air} / n$ and thus $\delta = 2\pi n\, \Delta x/\lambda_{air}$. The phase difference between the beams is $\Delta\delta = \delta_1 - \delta_2 = 2\pi n_1\, \Delta x/\lambda_{air} - 2\pi n_2\, \Delta x/\lambda_{air} = (2\pi\, \Delta x/\lambda_{air})(n_1 - n_2)$, where Δx is the thickness of the samples. No light is reaching the observer, so the beams are destructively interfering and thus $\Delta\delta = \pi$. We have $\pi = (2\pi\, \Delta x/\lambda_{air})(n_1 - n_2)$, or $\Delta x = 0.5\, \lambda_{air} / (n_1 - n_2) = (0.5)(650\text{ nm})/0.2 =$ 1625 nm.

3) (8 points) You are standing at the water's edge on Lake Michigan. The rising sun is creating a lot of glare on the water. If your eyes are 170 cm above the ground, how far away from the shore does a point on the water need to be in order for the light reflecting from that point to be 100% polarized when you look at it? The index of refraction of water is $^4/_3$.

Solution

The polarizing angle for reflection off any refracting surface is given by $\tan\theta = n$. In this case we have $\theta = \arctan(1.333) = 53°$. However, this angle is measured from the normal, so the angle between the water and the reflected ray is $90° – 53° = 37°$. Since your eyes are 170 cm high, a ray reflecting at 37° must satisfy $\tan(37°) = 1.7 / d$, where d is the distance to the shore in meters. We have $d = 1.7/\tan(37°) = 2.27$ m.

4) (12 points) You have a digital camera in which one of the lenses, L_2, is at a fixed distance of 3.5 cm from the CCD detector. With only this lens in the camera, an object that is 5.25 cm from L_2 can be perfectly focused on the detector. However, to photograph objects at other distances, a second lens L_1 (which is identical to lens L_2 in every way) must be placed in front of L_2. You notice that someone has left this distance set at x = 1 cm. How far from L_1 was the object that the previous user was trying to photograph? (It would be wise to explain your reasoning to gain potential partial credit.)



Solution

We want the distance that an object must be from L_1 in order to form an image 3.5 cm to the right of L_2. First we use $^1/_O + {}^1/_I = {}^1/_f$ to find the focal length of L_2 (and L_1): $^1/_{5.25} + {}^1/_{3.5} = {}^1/_f$, or f = 2.1 cm. Next we skip a lot of algebra by realizing that if 5.25 cm is the object distance for L_2 when the object is physical, then it is *still* the object distance when the object is an image. Therefore, since the object for L_2 is also the image from L_1, the L_1 image must be 5.25 cm – 1 cm = 4.25 cm to the left of L_1. This is a negative image distance relative to L_1. The object distance from L_1 is $^1/_O - {}^1/_{4.25} = {}^1/_{2.1}$, or O = 1.4 cm.

5) (10 points) I have two laser pointers, a red one with λ = 633 nm, and a green one with λ = 532 nm. Then I shine them on two slits that are separated by 7900 nm. How far (in cm) will the first red spot from the center of a screen four meters away be separated from the first green spot?

Solution
Since we are interested in the first two spots, we have m = 1. The distance y along the screen to either spot will satisfy $\tan\theta = y/(400 \text{ cm})$, and of course $d \sin\theta = m\lambda$. For the red spot, we quickly find that $\theta = \arcsin(633/7900) = 0.0802$, which is so small that we can reasonably set $\theta = \sin\theta = \tan\theta$ and save ourselves some algebra. We have 0.0802 = y/400 for the red spot, or y = 32.1 cm.

Likewise, for the green spot, we have θ = 532/7900 = 0.0673, so y = (0.0673)(400) = 26.9 cm. The distance between the spots is then 32.1 – 26.9 = 5.2 cm.