Midterm II, Phyx 135-3, Spring 2018 **Solution Key**

1) (10 points) You are travelling to Alpha Centauri in a star ship that is moving at 0.80 c relative to Earth. Your twin brother is also headed to Alpha Centauri, in a second star ship moving at 0.90 c relative to Earth and directly following you. As it happens, both of your watches were exactly synchronized when your twin brother's star ship was exactly one light-hour distant (behind yours) from your point of view. When your brother's star ship pulls even with yours, by how much will the times on the two watches differ?

Solution

We first need to calculate how fast your brother is moving relative to you. Since your brother is following you, we must subtract the given velocities like so: $v = (0.9\ c – 0.8\ c) / [1 – (0.8)(0.9)] = 0.3571\ c$.

Thus, from your point of view, your brother will cover the light-hour distance between you and him in $t = d/v = 1$ c-hr / 0.3571 c = 2.8 hours. However, the time that will run off his watch must be dilated from your point of view into 2.8 hours. In other words, we have 2.8 hrs = $\Delta t / (1 – 0.3571^2)^{1/2}$, where Δt is the "slow" time that you see on his watch. We have $\Delta t = (2.8)(0.934) = 2.615$ hrs. The difference between the two watches is 2.8 – 2.615 = 0.185 hr = 11.1 min.

2) (10 points) A soap film with an index of refraction $n = 1.40$ is hanging suspended in air. White light is shining on the film, but only light that has $\lambda = 602$ nm is being transmitted through the film with 100% efficiency. What is the minimum thickness of the film? **Note** – to obtain full credit on this problem, you must indicate your logic. A bare equation will not suffice.

Solution

For 100% of the light to shine through the film, there must no light reflected at that wavelength. In other words, we want the reflected waves to be fully destructive. There will be a phase flip at the first interface ($1.00 < 1.40$), but there will not be a phase flip at the second interface ($1.40 > 1.00$). So, if the two reflected waves are 2π out of phase, they will destructively interfere when one includes the phase flip.

We have $\delta = 2\pi\ \Delta x/\lambda$. In this case $\Delta x = 2L$, where L is the thickness of the soap film, and $\lambda = \lambda_{air}/n$. Inserting numbers, $2\pi = 2\pi\ (2L)(1.4)/602$ nm, or $L = (602\text{ nm})/2.8 =$ 215 nm.

3) (10 points) The muon is an unstable subatomic particle which has a mass of 105.66 MeV/c^2 and a lifetime of 2.197 x 10^{-6} s. Suppose you created a beam of muons here in Evanston and fired them through an underground tunnel to Los Angeles, 3200 km away. What is about the minimum kinetic energy the muons would need to have to reach Los Angeles?

Solution

This problem calls mostly for logic, not high mathematics. Even moving at the speed of light, a muon can only travel (3 x 10^8)(2.197 x 10^{-6}) = 659.1 m within its lifetime, which is a bit short of 3200 km. So, we see that we must time-dilate its lifetime by at least 3,200,000 / 659.1 = 4855 times to reach LA. Since the relativistic time-dilation formula and $E = mc^2$ use the same γ-factor, this means that we must raise its total energy to at least (4855)(105.66) = 513 GeV. Now, $E_K = E_{total} - m_o$, but 513 GeV is $\gg$ 106 MeV, so we can neglect the rest mass, and thus $E_K \approx$ 513 GeV.

P.S. -- With a γ-factor of 4855, one does not need to worry about our approximation that the muon is moving at the speed of light. If you care to calculate it out, γ = 4855 corresponds to v = 0.999999979 c.

4) The radius of a hydrogen atom, also known as the Bohr Radius, is about 5.29×10^{-11} m. The minimum wavelength of E&M radiation that a real hydrogen atom can emit is about 91 nm. The maximum is almost infinitely long. But, suppose you were to model this atom as an infinitely deep one-dimensional box.
4a) (4 points) What minimum wavelength of E&M radiation could the box emit? Briefly explain why.

Solution
Since the box is infinitely deep, it can have energy jumps that are infinitely high, that is, with infinite energy. But infinite energy corresponds to zero wavelength, so the box effectively has no minimum wavelength.

4b) (6 points) Calculate the maximum wavelength of E&M radiation the box could emit. (The mass of the electron is 9.11×10^{-31} kg.)

Solution
The maximum wavelength will be given by the smallest energy jump, which will be from n = 2 to n = 1. We have $E = n^2h^2 / 8mL^2 = (1)^2(6.626 \times 10^{-34})^2 / 8(9.11 \times 10^{-31})(4)(5.29 \times 10^{-11})^2 = 33.6$ eV for the first energy level in the box. The second energy level will be $(2)^2(33.6) = 134.4$ eV. The difference in energy is thus 134.4 – 33.6 = 100.8 eV. The corresponding wavelength is $\lambda = 1240 / 100.8 =$ 12.3 nm.

5) (10 points) You have a solar cell which is 15 cm long and 5 cm wide. The cell is made of a material which has a work function of 2.1 eV. Bright noon-time sunlight is shining on the cell, and the cell is producing 1.3 W of electric power. What is the efficiency of the solar cell, in terms of electrons produced per input photon?

Intensity of bright sunlight at noon: 1300 W/m^2
Wavelength of sunlight you can assume: 501 nm
1 amp = 6.24×10^{18} electrons / s
Electric Power, P = IV

Solution

The power hitting the cell is (1300)(0.15)(0.05) = 9.75 W. The energy of the incoming photons is E = 1240 / 501 = 2.475 eV = 3.965×10^{-19} J, so that means we have 9.75 / (3.965×10^{-19}) = 2.46×10^{19} photons/s striking the cell.

Now, the energy of the emitted electrons must be 2.475 – 2.1 = 0.375 eV. Or to put it another way, the voltage in the circuit is 0.375 volts. We therefore have 1.3 W = I(0.375), or I = 3.467 amps = (3.467)(6.24×10^{18}) = 2.16×10^{19} electrons/s.

The cell's efficiency is 2.16/2.46 = 87.8%