Phyx 135-3 (1 pm), Winter 2016 **Name ___________________________**

1a) (7 points) A beam of electrons is directed at two slits which are 4 μm apart. On a fluorescent screen four meters away, they form an interference pattern where the center of the first side-fringe is 0.2 mm from the center of the central bright spot. What is the kinetic energy in eV of the electrons? (You may assume non-relativistic electrons. The electron mass is 9.11×10^{-31} kg.)

Solution

We have $d \sin\theta = m\lambda$ for the maxima resulting from simple two-slit interference, and in this case $\sin\theta \approx \theta = 0.2 \text{ mm} / 4 \text{ m} = 5 \times 10^{-5}$. So, $\lambda = (4 \times 10^{-6})(5 \times 10^{-5}) = 2 \times 10^{-10}$ m for the electrons.

We know that $p = h/\lambda$, and $E = ½ mv^2 = ½(mv)^2/m = p^2/2m$, so $E = h^2/2m\lambda^2 =$
$(6.626 \times 10^{-34})^2 / 2(9.11 \times 10^{-31})(2 \times 10^{-10})^2 = 6.024 \times 10^{-18}$ J = 37.65 eV

1b) (3 points) We *assumed* non-relativistic electrons for part 1a. Using your answer for part 1a, give a *brief* explanation for why this was or was not a good assumption.

Answer

The kinetic energy of the electrons (37.65 eV) is quite small compared to the electron rest mass of 511 keV. Thus γ is small and our assumption of non-relativistic physics is good.

Alternatively, one could note that $mv = h/\lambda$, so $v = (6.626 \times 10^{-34})/(9.11 \times 10^{-31})(2 \times 10^{-10}) =$
3.64×10^6 m/s, which is barely 1% the speed of light. This is non-relativistic.

2) A passenger aboard a spaceliner is furious that the ship's breakfast buffet has no onion bagels. The spaceliner happens to be close to Earth, so the crew asks an Earth deli to immediately send them an onion bagel by special courier.

2a) (4 points) If the spaceliner is moving towards the Earth at 0.8c and the special courier is moving away from the Earth at 0.85c (they are moving in straight lines towards each other), how fast is the special courier moving relative to the spaceliner?

Solution
We use the relativistic velocity addition formula where $v = (U + V) / (1 + UV/c^2)$. Using $U = 0.8c$ and $V = 0.85c$ gives us $v = 1.65c / (1 + 0.68) =$ 0.982c

2b) (6 points) The Earth deli decides that the energy needed to accelerate the onion bagel to a speed of 0.85c must be added to the bill for the spaceliner. Assuming a cost of 12 cents per kilowatt-hour for the electricity, how much will it cost the spaceliner to purchase a 30-g bagel? (One kw-hr = 3,600,000 J.)

Solution
The kinetic energy of a relativistic object is given by $\gamma\, m_o c^2 - m_o c^2$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. In this case $\gamma = (1 - 0.85^2)^{-1/2} = 1.898$, and $m_o c^2 = (0.03)(3 \times 10^8)^2 = 2.7 \times 10^{15}$ J. So, the energy needed is $(1.898 - 1)(2.7 \times 10^{15})/(3.6 \times 10^6) = 6.735 \times 10^8$ kw-hr. At 0.12 dollars per kw-hr, we have a final energy cost of $80,800,000

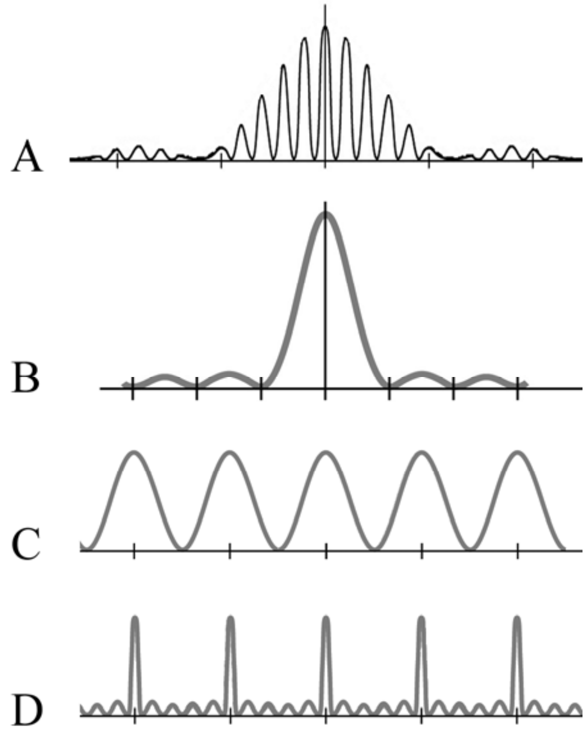
3) (3 points each) Four intensity patterns are shown at right. Match the correct pattern with the relevant description below.

_B_ I Light passing through a single narrow slit.

_D__ II Light passing through a grating with 10 slits.

_C__ III Two waves (light or sound) of equal frequency and amplitude radiating spherically from two point sources.

_A___ IV Monochromatic light passing through two narrow slits set very close together.



4) (10 points) A relativistic radon atom is approaching the Earth at 0.90 c when it emits a γ-ray (towards the Earth) which has an energy of 186 keV in the rest frame of the radon. What is the energy of the γ-ray in the rest frame of the Earth?

Solution

An astonishing number of students tried to solve this problem by adding the speed of the γ-ray to that of the radon atom, or by attempting to relativistically dilate the energy of the γ-ray, or by using some other relativistic kinematic formula.

All of these approaches are incorrect, because of course the speed of the γ-ray photon never varies from any viewpoint. However, from E = hf, we know that its *frequency* and therefore its energy can vary due to the Doppler effect.

So, using the relativistic Dopper shift formula, we know that the frequency of the radiation emitted by the radon will be increased (blue-shifted) relative to the Earth by a ratio of
$f / f_0 = [(1 + 0.9) / (1 – 0.9)]^{1/2} = 4.36$ times. Since photon energy is directly proportional to frequency, the photon's energy will be (186 keV)(4.36) = 811 keV.

5) I am shining three different lasers of three different colors onto a sheet of metal: a blue laser (λ = 420 nm), a green laser (λ = 532 nm), and a red laser (λ = 620 nm). I have noticed that electrons with a kinetic energy of 0.40 eV (only that energy) are being emitted by the metal.

5a) (3 points) Which color of laser is producing the observed electrons? ________

Answer: If only one energy of electron is coming off, then only the laser with the *highest* photon energy is above the critical frequency. That would have to be the laser with the *shortest* wavelength, i.e., the blue laser.

5b) (7 points) What is the work function (in eV) of this particular metal?

Solution

We know that $E_k = hf - \Phi$. The photon energy for the blue laser can be quickly calculated using the short-cut formula $E(\text{eV}) = 1240 / \lambda(\text{nm}) = 1240/420 = 2.95$ eV, so we have $0.40 = 2.95 - \Phi$, or

$\Phi = 2.55$ eV

6) (8 points) Suppose you have a quantum system with the following wave function:

$\psi(x) = (1.427)\, e^{-x}$ for $0 \le x \le 2$,

and $\psi = 0$ everywhere else.

What is the probability that you will find this particle somewhere between x = 0 and x = 1?

Solution

The total probability will be given by the integral from 0 to 1 of $\psi^2 = 2.036\, e^{-2x}$. We have:
$(2.036)(-½)e^{-2x}$ evaluated from 1 to 0 = $-1.018(e^{-2} - 1)$ = 88%