Physics 135-3, Quiz #1 Solutions

1) Two waves are moving along the x-axis in the positive direction. The origin (zero coordinate) for their position and time are the same. They are given by: $y_1(x, t) = 2 \cos(5 x - 32 t)$, and $y_2(x, t) = 3 \cos(4 x - 24 t)$, where all units are in meters and seconds.

1a) (3 points) What is the wavelength of y_1?
1b) (3 points) What is the frequency of y_2?
1c) (3 points) At t = 0, what is the phase (in degrees) of y_1 at a distance of 12 m from the origin?
1d) (3 points) What is the speed of each wave?

1a) Remembering that a sinusoidal wave can be written as $A \cos(kx - \omega t)$, we have $k = 5 = 2\pi/\lambda$, or $\lambda = 2\pi/5 =$ 1.257 meter.
1b) We have $\omega = 24 = 2\pi f$, or $f = 24 / 2\pi =$ 3.82 Hz.
1c) $\Delta\delta = 2\pi \Delta x / \lambda = 2\pi$ (12 m) / (1.257 m) $= 2\pi(9.547)$, of which 9 cycles do not count because they represent full wavelengths. We have $\Delta\delta = 2\pi(0.547)$ rad = 197°.
1d) $v = \omega/k = 32 / 5 =$ 6.4 m/s for y_1, and $24 / 4 =$ 6 m/s for y_2.

2) (8 points) You have a guitar. String one has $\mu = 10$ g/m and string two has $\mu = 5.5$ g/m. String two has a tension of 200 N on it, but string one has come loose. The strings are the same length. However, you know that if you finger string one at a point ¼ of the way down from the guitar head, then string one will be at the same pitch as an unfingered string two. What tension do you need to apply to string one? (Hint: don't be afraid to use algebra, and use ratios.)

Solution
The speed of a wave on a string is $v = (F/\mu)^{1/2}$, so its frequency must be $f = v/\lambda = (1/\lambda)(F/\mu)^{1/2}$. We are told that $f_1 = f_2$ when λ_1 is fingered ¼ of the way down, or in other words the ratio $f_1 / f_2 = 1$ when $\lambda_1 = \frac{3}{4}\lambda_2$. Taking the ratio and rearranging: $f_1 / f_2 = [(1/\lambda_1)(F_1/\mu_1)^{1/2}] / [(1/\lambda_2)(F_2/\mu_2)^{1/2}] = (\frac{3}{4}\lambda_2)^{-1}(\lambda_2)(F_1/\mu_1)^{1/2} (\mu_2/F_2)^{1/2} = (^4/_3)(F_1/F_2)^{1/2} (5.5 /10)^{1/2} = 1$. We have $F_1 = (^3/_4)^2(10/5.5)(200 \text{ N}) =$ 204.5 N.