Physics 135-3, Quiz #3 Solutions

1) (10 points) An oil slick (n = 1.45) is floating on a puddle of water (n = 1.33). It is reflecting light at two wavelengths, λ = 690 nm and λ = 414 nm. How thick is the oil slick?

Solution

We note that light reflecting from the oil will undergo a phase flip, whereas light reflecting from the water will not. We are given two wavelengths at which constructive interference occurs, and that would normally require $\Delta\delta = 2\pi$. However, since we have a phase flip, we instead require that $\Delta\delta = m\pi$, where m is an odd integer.

We next use $\Delta\delta = 2\pi\ \Delta x/\lambda = 2\pi n(2L)/\lambda_{air}$, where n is the index of refraction for the oil, and L is the thickness of the film. Since we have two λ, we have two equations: $m_{690}\,\pi = 2\pi(1.45)2L/690$ nm, and $m_{414}\,\pi = 2\pi(1.45)2L/414$ nm. The two m's must be odd integers, so let us divide the first equation by the second to get: $m_{690} / m_{414} = 414 / 690 = 0.60$ exactly, which is $^3/_5$. We can use either m to calculate L, so let's use $m_{690} = 3$ for λ = 690 nm. We have: $3 = (2.9)(2L) / 690$ nm, or L = 356.9 nm.

2) (10 points) The opening to the human eye, under very dark conditions, is about 8 mm across. Suppose someone is holding two candles in their hand, which is to say, the two flames are about 2.5 cm apart. At what minimum distance would they need to stand such that you could no longer tell if they were holding one candle or two with your naked eye? (You may assume that yellow light has a wavelength of 567 nm.)

Solution

We can use the Rayleigh Criterion, which says $\Delta\theta = 1.22\ \lambda/a$. We also have $\Delta\theta \approx \tan(\Delta\theta) = x/d$, where x is the distance between the two candles, and d is the distance to the candles. This yields $x/d = 1.22\ \lambda/a$, or $d = xa/1.22\ \lambda = (0.025)(0.008)/1.22(5.67 \times 10^{-7}) =$ 289 m.