Physics 135-3, Quiz #4 Solutions

1) (12 points) You are watching a space battle from a bunker on an asteroid. An evil space cruiser moving at 0.60 c relative to you is being chased by an intrepid space fighter moving at 0.80 c, also relative to you. Then, the space cruiser fires an anti-good-guys missile directly at the space fighter. You happen to know that AGG missiles always travel at 0.70 c relative to the ship that fires them. What what speed does the space fighter's pilot see the missile approaching him?

Solution

There are several ways to do this problem, but they all include using the formula for relativistic velocity addition twice. I will start by calculating how fast the intrepid fighter is approaching the evil cruiser from the fighter pilot's point of view. Using Newtonian physics, we know that the relative speed between the two would be 0.8 c – 0.6 c = 0.2 c, or in other words, we know that one of the velocities has to be negative. This will also be true using Einstein's physics. So, we have: $v = (0.8\ c - 0.6\ c) / [1 - (0.8c)(0.6\ c) / c^2] = 0.2\ c / 0.52 = 0.3846\ c$. Thus the pilot sees himself as standing still, and the evil cruiser as "backing up" towards him at 0.3846 c.

We next need to add this relative speed to the speed of the AGG missile. We have:
$v = (0.3846\ c + 0.70\ c) / [1 + (0.3846\ c)(0.70\ c) / c^2] = 1.0846\ c / 1.2692 =$ 0.855 c.

2) (8 points) Let us suppose that a certain subatomic particle with a rest mass of 112 MeV/c^2 is known to spontaneously emit γ-rays with $\lambda = 1.20 \times 10^{-12}$ m. If this particle were to be shot directly away from you with a kinetic energy of 28 MeV, what would be the wavelength of the γ-rays reaching you then?

Solution

The γ-rays will change wavelength because they are being Doppler-shifted. Since the particle is moving away from you, you need to use the red-shift formula, $f = f_o[(1 - \beta)/(1 + \beta)]^{1/2}$. We can find β by using $E = mc^2$. We know that $m_oc^2 = 112$ MeV, and E = 112 + 28 = 140 MeV, so we have: $140 = 112 / (1 - \beta^2)^{1/2}$, or $1 - \beta^2 = (112/140)^2 = 0.64$, or $0.60 = \beta$.

Putting this into the Doppler formula yields $f = c/\lambda = [c/(1.20 \times 10^{-12})][(1 - 0.6)/(1 + 0.6)]^{1/2}$, or
$\lambda = (1.20 \times 10^{-12}) / (0.4 / 1.6)^{1/2} = 2.40 \times 10^{-12}$ m.