Physics 135-3, Quiz #5 **Solution Key**

1) (14 points) A certain spectral line from the very rare element Hypotheticalium has a wavelength of λ = 584 nm. You notice this line in the spectrum of a distant star, but you also notice that it is flanked by lines at λ = 583.8 nm and λ = 584.2 nm. What does this tell you about the strength of the magnetic field on the surface of that distant star? ($\mu_B = 9.274 \times 10^{-24}$ J/T)

Solution

The Zeeman effect has caused this line to split into a triplet. We have $\Delta U = m\, \mu_B\, B$, where m is clearly one in this case, and ΔU is given by the difference in energy between the lines. Using the short-cut formula E(eV) = 1240/λ(nm), we have: E_1 = 1240/584 = 2.12329 eV, and E_2 = 1240 / 584.2 = 2.12256 eV. The difference is ΔU = 2.12329 – 2.12256 = 0.00073 eV = 1.17×10^{-22} J. Then $B = (1.17 \times 10^{-22})/(9.274 \times 10^{-24})$ = 12.6 T

2) (6 points) The electron is said to be "spin ½", which is only a jargony way of saying that it has a fixed intrinsic angular momentum quantum number. Suppose you were to pass an electron through a magnetic field. What are the only two angles that the electron's angular momentum vector could make with the magnetic field?

Solution

Angular momentum is given by $L = [l(l + 1)]^{1/2}\hbar$. For the intrinsic angular momentum of the electron, l = ½, so $L = [½ (½ + 1)]^{1/2}\hbar = ½\sqrt{3}\, \hbar$. The z-component of angular momentum is given by $L_z = m\hbar$, and $-l \leq m \leq l$ with m integer, so m can only be $\pm$½. We therefore have only two possible orientations between the magnetic field (or L_z) and L: $\cos\theta = L_z / L = (½\hbar) / (½\sqrt{3}\, \hbar) = 1/\sqrt{3}$, or θ = 54.74°, and the symmetric angle to this one, 180° – 54.74° = 125.26°