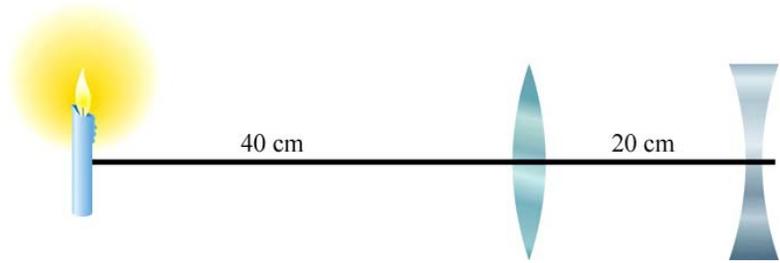


1) A candle is 40 cm to the left of a convex lens, which in turn is 20 cm to the left of a concave lens. The *magnitude* of the focal length of both lenses is 50 cm.



1a) (8 points) If the candle is 10 cm tall, how tall is the image of the candle created by both lenses?

Solution

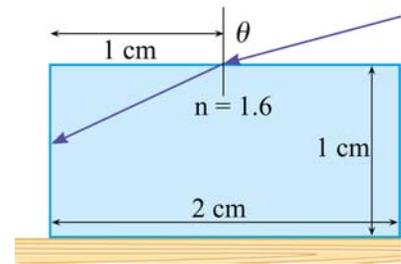
We start with the formula $\frac{1}{f} = \frac{1}{O} + \frac{1}{I}$. For the first lens, $f = +50$ cm and $O = +40$ cm, so we have: $\frac{1}{I} = \frac{1}{50} - \frac{1}{40}$, or $I = -200$ cm. This places the image 200 cm to the left of the first lens, which means it is 220 cm to the left of the second lens. The image of the first lens becomes the object of the second, so we again apply $\frac{1}{f} = \frac{1}{O} + \frac{1}{I}$, but this time with $f = -50$ cm (for a diverging lens). We have: $-\frac{1}{50} - \frac{1}{220} = \frac{1}{I}$, or $I = -40.74$ cm.

The magnification of the first lens is $m = -(-200)/40 = +5$, and the magnification of the second lens is $m = -(-40.74)/220 = +0.185$, so the overall magnification is $(5)(0.185) = +0.926$. The image of the candle is thus **9.26 cm** high.

1b) (2 points) Is the image inverted or right side up?

Answer: The overall magnification is positive, so the image is right side up.

2) A ray of light is shining on the top center of a glass block. It can be at any angle θ . The block has an index of refraction of $n = 1.6$, and is 2 cm wide and 1 cm high. In the following questions, you must support your answer with calculations and/or diagrams. Simple guesses or hunches will not be given credit.



2a) (5 points) Suppose the bottom surface of the block is coated with a black layer that absorbs all the light hitting it. Is it possible for any incoming light to escape through either side of the block?

Solution

The maximum possible angle at which light can hit the center point is 90° . So, the maximum possible angle for refracted light inside the glass is $(1.00) \sin(90^\circ) = (1.6) \sin\theta_2$, or $\theta_2 = 38.7^\circ$. However, the bottom left (or right) corner is 1 cm down and 1 cm over from the entry point, which corresponds to 45° . The refracted light cannot achieve this angle, so all of it will hit the bottom and be absorbed. The answer is no.

2b) (5 points) Suppose the bottom surface of the block is coated with a perfectly reflecting metallic film. Is it possible for any incoming light to escape through either side of the block?

Solution

If light from the entry point hits the bottom $\frac{1}{2}$ cm from the side, it can reflect upwards and just touch the top left (or right) corner of the block. This corresponds to $\tan\theta = (\frac{1}{2} \text{ cm}) / (1 \text{ cm})$, or $\theta = 26.6^\circ$. From Part 2a we know that light can hit the bottom at 38.7° , so it would seem possible for reflected light to escape through the sides of the block. *But wait! There's more!* A maximum angle of reflection of 38.7° from the bottom means that the minimum angle at which the light can strike the sides is $90^\circ - 38.7^\circ = 51.3^\circ$. Snell's Law then gives us $(1.6) \sin(51.3^\circ) = (1.00) \sin\theta_2$, or $\sin\theta_2 = 1.25$, which is impossible! The light will experience total internal reflection and bounce away. None will escape through the sides.

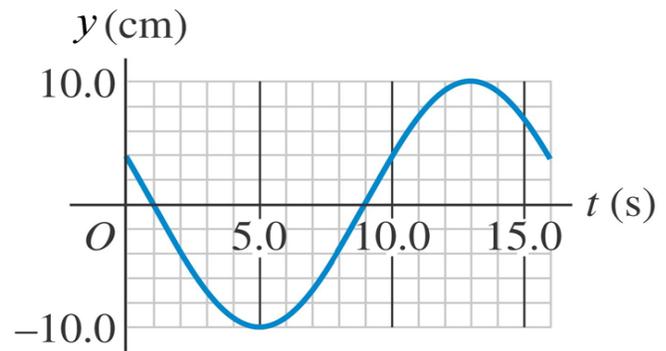
3) (10 points) You are enjoying a roller coaster ride at the Iowa state fair when suddenly an insane stuntman who is playing a violin while he stands on the wings of a biplane roars past you. (That is, you are both going the same direction, with the stuntman in front.) You also notice that the 'A' string on his violin sounds a bit flat. You hear 400 Hz for the 'A' string, whereas a properly tuned violin must have the 'A' at 440 Hz. You remember reading someplace that stunt biplanes always fly at 179.78 miles per hour (= 80 m/s), so you decide to use that fact to calculate the speed of the roller coaster you are riding in. And – what is that speed? (Assume $c = 343$ m/s.)

Solution

The general Doppler equation is $f = f_0 (v \pm v_{\text{obs}}) / (v \pm v_{\text{tone}})$. In this case the observer is moving towards the source, tending to increase the frequency, so v_{obs} must be positive in the numerator. The source is moving away from the observer, tending to lower the frequency, so v_{tone} must be positive in the denominator. We have $f_0 = 440$ Hz, $f = 400$ Hz, $v = 343$ m/s, and $v_{\text{tone}} = 80$ m/s. Inserting everything into the equation:

$$400 = 440(343 + v_{\text{obs}}) / (343 + 80), \text{ or } v_{\text{obs}} = (10/11)(423) - 343 = \mathbf{41.5 \text{ m/s.}}$$

4) A wave travelling down a wire is described by the graph at the right. The wire is under a tension of 2500 N, and has a linear density of 12 g/m.



4a) (2 points) What is the period of this wave?

Answer: 16 s, as read off the graph.

4b) (2 points) What is the velocity of this wave?

Answer: Using $v = (F/\mu)^{1/2}$ gives us $v = (2500 / 0.012)^{1/2} = 456.4 \text{ m/s}$

4c) (3 points) What is the wave number k of this wave?

Answer: $k = 2\pi/\lambda$, and $\lambda = v/f = vT = (456.4)(16) = 7302 \text{ m}$, so $k = 2\pi/7302 = 0.00086 \text{ m}^{-1}$

4d) (3 points) Where is the wave (what is the value of x) when $t = 0$?

Solution

The wave has an amplitude of 4 cm at $t = 0$, i.e., $\sin(kx - \omega t) = 0.4$ at $t = 0$. The inverse sin of 0.4 is 0.4115 radians, so $x = 0.4115 / 0.00086 = 478.5 \text{ m}$

Multiple Choice / Short Answer

___C___ 5) (2 points) The magnification of a simple refracting telescope is given by:

- A) The object lens diameter divided by the eyepiece lens diameter
- B) The eyepiece lens diameter divided by the object lens diameter
- C) The object lens focal length divided by the eyepiece lens focal length
- D) The eyepiece lens focal length divided by the object lens focal length
- E) The object distance divided by the object lens image distance
- F) The object lens image distance divided by the object distance

6) (3 points) If I am shouting with a power of 0.025 watts, and you are standing 4 m away, what decibel level are you experiencing?

Solution: The power at 4 meters is $0.025 / 4\pi 4^2 = 1.24 \times 10^{-4} \text{ W/m}^2$. This translates into:
 $\beta = (10 \text{ dB}) \log(1.24 \times 10^{-4} / 10^{-12}) = 80.9 \text{ dB}$

___B___ 7a) (2 points) The longest pipes on a pipe organ are:

- A) Open on both ends
- B) Open on one end, closed on the other
- C) Closed on both ends

7b) (3 points) Why? (Give a brief explanation for your answer to 7a)

Half-open pipes sound an octave lower than open-ended pipes, so the half-open pipes are used because they are shorter (one-half the length) than open-ended pipes sounding the same note. It saves space and money, and lets the shorter pipes fit underneath the ceiling.