

4. The distance d between the beetle and the scorpion is related to the transverse speed v_t and longitudinal speed v_ℓ as

$$d = v_t t_t = v_\ell t_\ell$$

where t_t and t_ℓ are the arrival times of the wave in the transverse and longitudinal directions, respectively. With $v_t = 50$ m/s and $v_\ell = 150$ m/s, we have

$$\frac{t_t}{t_\ell} = \frac{v_\ell}{v_t} = \frac{150 \text{ m/s}}{50 \text{ m/s}} = 3.0.$$

Thus, if

$$\Delta t = t_t - t_\ell = 3.0t_\ell - t_\ell = 2.0t_\ell = 4.0 \times 10^{-3} \text{ s} \Rightarrow t_\ell = 2.0 \times 10^{-3} \text{ s},$$

then $d = v_\ell t_\ell = (150 \text{ m/s})(2.0 \times 10^{-3} \text{ s}) = 0.30 \text{ m} = 30 \text{ cm}$.

7. (a) Recalling from Chapter 12 the simple harmonic motion relation $u_m = y_m \omega$, we have

$$\omega = \frac{16}{0.040} = 400 \text{ rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = 80/64 = 1.26 \text{ m} \approx 1.3 \text{ m}$.

(c) The amplitude of the transverse displacement is $y_m = 4.0 \text{ cm} = 4.0 \times 10^{-2} \text{ m}$.

(d) The wave number is $k = 2\pi/\lambda = 5.0 \text{ rad/m}$.

(e) The angular frequency, as obtained in part (a), is $\omega = 16/0.040 = 4.0 \times 10^2 \text{ rad/s}$.

(f) The function describing the wave can be written as

$$y = 0.040 \sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.040$ at $x = t = 0$. Therefore, $\sin \phi = 1$, for which the “simplest” root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040 \sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of ω is minus.

10. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$. $\lambda = 1.0 \times 10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0 \times 10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\max} = 2\pi f y_m = (4.0 \pi \text{ s}^{-1})(6.0 \text{ cm}) = 75 \text{ cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0 \text{ cm}$.

12. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t).$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi \sqrt{15^2 - y^2}.$$

Therefore, where $y = 12$, u must be $\pm 135\pi$. Consequently, the *speed* there is $424 \text{ cm/s} = 4.24 \text{ m/s}$.