1. The magnitude L of the orbital angular momentum $\vec{L}$ is given by Eq. 40-2: $L=\sqrt{\ell(\ell+1)}\hbar$. On the other hand, the components L_z are $L_z=m_\ell\hbar$, where $m_\ell=-\ell,...+\ell$. Thus, the semi-classical angle is $\cos\theta=L_z/L$. The angle is the smallest when $m=\ell$, or

$$\cos\theta=\frac{\ell\hbar}{\sqrt{\ell(\ell+1)}\hbar}\quad\Rightarrow\quad\theta=\cos^{-1}\left(\frac{\ell}{\sqrt{\ell(\ell+1)}}\right)$$

With $\ell=5$, we have $\theta=\cos^{-1}(5/\sqrt{30})=24.1°$.

2. For a given quantum number n there are n possible values of ℓ, ranging from 0 to $n-1$. For each ℓ the number of possible electron states is $N_\ell = 2(2\ell+1)$. Thus the total number of possible electron states for a given n is

$$N_n=\sum_{\ell=0}^{n-1}N_\ell=2\sum_{\ell=0}^{n-1}(2\ell+1)=2n^2.$$

Thus, in this problem, the total number of electron states is $N_n = 2n^2 = 2(5)^2 = 50$.

3. (a) We use Eq. 40-2:

$$L=\sqrt{\ell(\ell+1)}\hbar=\sqrt{3(3+1)}\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)=3.65\times10^{-34}\text{ J}\cdot\text{s}.$$

(b) We use Eq. 40-7: $L_z=m_\ell\hbar$. For the maximum value of L_z set $m_\ell=\ell$. Thus

$$\left[L_z\right]_{\max}=\ell\hbar=3\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)=3.16\times10^{-34}\text{ J}\cdot\text{s}.$$

10. (a) For $n = 3$ there are 3 possible values of ℓ: 0, 1, and 2.

(b) We interpret this as asking for the number of distinct values for m_ℓ (this ignores the multiplicity of any particular value). For each ℓ there are $2\ell+1$ possible values of m_ℓ. Thus the number of possible m_ℓ's for $\ell = 2$ is $(2\ell + 1) = 5$. Examining the $\ell = 1$ and $\ell=0$ cases cannot lead to any new (distinct) values for m_ℓ, so the answer is 5.

(c) Regardless of the values of n, ℓ and m_ℓ, for an electron there are always two possible values of m_s: $\pm\frac{1}{2}$.

(d) The population in the $n = 3$ shell is equal to the number of electron states in the shell, or $2n^2 = 2(3^2) = 18$.

(e) Each subshell has its own value of ℓ. Since there are three different values of ℓ for n = 3, there are three subshells in the n = 3 shell.