

27. (a) All states with principal quantum number $n = 1$ are filled. The next lowest states have $n = 2$. The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same. Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}$ or $n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}$. That is, $(n, \ell, m_\ell, m_s) = (2, 0, 0, +1/2)$ and $(2, 0, 0, -1/2)$.

(b) The next lowest state in energy is an $n = 2, \ell = 1$ state. All $n = 3$ states are higher in energy. The magnetic quantum number can be $m_\ell = -1, 0,$ or $+1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Thus, $(n, \ell, m_\ell, m_s) = (2, 1, 1, +1/2), (2, 1, 1, -1/2), (2, 1, 0, +1/2), (2, 1, 0, -1/2), (2, 1, -1, +1/2)$ and $(2, 1, -1, -1/2)$.

28. For a given value of the principal quantum number n , there are n possible values of the orbital quantum number ℓ , ranging from 0 to $n - 1$. For any value of ℓ , there are $2\ell + 1$ possible values of the magnetic quantum number m_ℓ , ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ , there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2 \sum_{\ell=0}^{n-1} (2\ell + 1).$$

Now

$$\sum_{\ell=0}^{n-1} 2\ell = 2 \sum_{\ell=0}^{n-1} \ell = 2 \frac{n}{2} (n-1) = n(n-1),$$

since there are n terms in the sum and the average term is $(n - 1)/2$. Furthermore,

$$\sum_{\ell=0}^{n-1} 1 = n.$$

Thus $N = 2[n(n-1) + n] = 2n^2$.

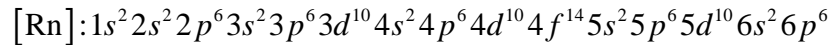
29. The total number of possible electron states for a given quantum number n is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2 \sum_{\ell=0}^{n-1} (2\ell + 1) = 2n^2.$$

Thus, if we ignore any electron-electron interaction, then with 110 electrons, we would have two electrons in the $n = 1$ shell, eight in the $n = 2$ shell, 18 in the $n = 3$ shell, 32 in the $n = 4$ shell, and the remaining 50 ($= 110 - 2 - 8 - 18 - 32$) in the $n = 5$ shell. The 50

electrons would be placed in the subshells in the order s, p, d, f, g, h, \dots and the resulting configuration is $5s^2 5p^6 5d^{10} 5f^{14} 5g^{18}$. Therefore, the spectroscopic notation for the quantum number ℓ of the last electron would be g .

Note, however, when the electron-electron interaction is considered, the ground-state electronic configuration of darmstadtium actually is $[\text{Rn}]5f^{14} 6d^9 7s^1$, where



represents the inner-shell electrons.

31. The first three shells ($n = 1$ through 3), which can accommodate a total of $2 + 8 + 18 = 28$ electrons, are completely filled. For selenium ($Z = 34$) there are still $34 - 28 = 6$ electrons left. Two of them go to the $4s$ subshell, leaving the remaining four in the highest occupied subshell, the $4p$ subshell.

- (a) The highest occupied subshell is $4p$.
- (b) There are four electrons in the $4p$ subshell.

For bromine ($Z = 35$) the highest occupied subshell is also the $4p$ subshell, which contains five electrons.

- (c) The highest occupied subshell is $4p$.
- (d) There are five electrons in the $4p$ subshell.

For krypton ($Z = 36$) the highest occupied subshell is also the $4p$ subshell, which now accommodates six electrons.

- (e) The highest occupied subshell is $4p$.
- (f) There are six electrons in the $4p$ subshell.