27. (a) All states with principal quantum number n = 1 are filled. The next lowest states have n = 2. The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same. Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}$ or $n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}$. That is, (n, ℓ, m_ℓ, m_s) = (2,0,0, +1/2) and (2,0,0, –1/2).

(b) The next lowest state in energy is an n = 2, $\ell = 1$ state. All n = 3 states are higher in energy. The magnetic quantum number can be $m_\ell = -1, 0, \text{ or } +1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Thus, (n, ℓ, m_ℓ, m_s) = (2,1,1, +1/2), (2,1,1, –1/2), $(2,1,0,+1/2)$, $(2,1,0,-1/2)$, $(2,1,-1,+1/2)$ and $(2,1,-1,-1/2)$.

28. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to n – 1. For any value of ℓ, there are $2\ell+1$ possible values of the magnetic quantum number m_ℓ, ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2\sum_{\ell=0}^{n-1}(2\ell+1).$$

Now

$$\sum_{\ell=0}^{n-1} 2\ell = 2\sum_{\ell=0}^{n-1}\ell = 2\frac{n}{2}(n-1) = n(n-1),$$

since there are n terms in the sum and the average term is $(n - 1)/2$. Furthermore,

$$\sum_{\ell=0}^{n-1} 1 = n\ .$$

Thus $N = 2\left[n(n-1)+n\right] = 2n^2$.

29. The total number of possible electron states for a given quantum number n is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2\sum_{\ell=0}^{n-1}(2\ell+1) = 2n^2.$$

Thus, if we ignore any electron-electron interaction, then with 110 electrons, we would have two electrons in the $n = 1$ shell, eight in the $n = 2$ shell, 18 in the $n = 3$ shell, 32 in the $n = 4$ shell, and the remaining 50 $(= 110 - 2 - 8 - 18 - 32)$ in the $n = 5$ shell. The 50 electrons would be placed in the subshells in the order $s, p, d, f, g, h, \ldots$ and the resulting configuration is $5s^2 5p^6 5d^{10} 5f^{14} 5g^{18}$. Therefore, the spectroscopic notation for the quantum number ℓ of the last electron would be *g*.

Note, however, when the electron-electron interaction is considered, the ground-state electronic configuration of darmstadtium actually is $[\mathrm{Rn}]5f^{14}6d^9 7s^1$, where

$$[\mathrm{Rn}]: 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$$

represents the inner-shell electrons.

31. The first three shells (n = 1 through 3), which can accommodate a total of 2 + 8 + 18 = 28 electrons, are completely filled. For selenium (Z = 34) there are still 34 – 28 = 6 electrons left. Two of them go to the 4*s* subshell, leaving the remaining four in the highest occupied subshell, the 4*p* subshell.

(a) The highest occupied subshell is 4*p*.

(b) There are four electrons in the 4*p* subshell.

For bromine (Z = 35) the highest occupied subshell is also the 4*p* subshell, which contains five electrons.

(c) The highest occupied subshell is 4*p*.

(d) There are five electrons in the 4*p* subshell.

For krypton (Z = 36) the highest occupied subshell is also the 4*p* subshell, which now accommodates six electrons.

(e) The highest occupied subshell is 4*p*.

(f) There are six electrons in the 4*p* subshell.