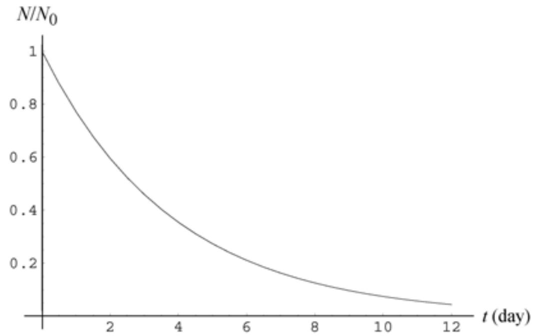
30. We note that t = 24 h is four times $T_{1/2}$ = 6.5 h. Thus, it has reduced by half, four-fold:

$$\left(\frac{1}{2}\right)^4\left(48\times10^{19}\right)=3.0\times10^{19}.$$

The fraction of the Hg sample remaining as a function of time (measured in days) is plotted below.



31. (a) The decay rate is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. Initially, $R = R_0 = \lambda N_0$, where N_0 is the number of undecayed nuclei at that time. One must find values for both N_0 and λ. The disintegration constant is related to the half-life $T_{1/2}$ by

$$\lambda=(\ln 2)/T_{1/2}=(\ln 2)/(78\text{h})=8.89\times10^{-3}\,\text{h}^{-1}.$$

If M is the mass of the sample and m is the mass of a single atom of gallium, then $N_0 = M/m$. Now,

$$m = (67\text{ u})(1.661\times10^{-24}\text{ g/u}) = 1.113\times10^{-22}\text{ g}$$

and

$$N_0 = (3.4\text{ g})/(1.113\times10^{-22}\text{ g}) = 3.05\times10^{22}.$$

Thus,

$$R_0 = (8.89\times10^{-3}\text{ h}^{-1})(3.05\times10^{22}) = 2.71\times10^{20}\text{ h}^{-1} = 7.53\times10^{16}\text{ s}^{-1}.$$

(b) The decay rate at any time t is given by

$$R = R_0\, e^{-\lambda t}$$

where R_0 is the decay rate at t = 0. At t = 48 h, $\lambda t = (8.89\times10^{-3}\text{ h}^{-1})(48\text{ h}) = 0.427$ and

$$R=\left(7.53\times10^{16}\text{ s}^{-1}\right)e^{-0.427}=4.91\times10^{16}\text{ s}^{-1}.$$

34. Combining Eqs. 42-20 and 42-21, we obtain

$$M_{\text{sam}} = N\frac{M_{\text{K}}}{M_{\text{A}}} = \left(\frac{RT_{1/2}}{\ln 2}\right)\left(\frac{40\text{ g / mol}}{6.02\times 10^{23}\text{ / mol}}\right)$$

which gives 0.66 g for the mass of the sample once we plug in 1.7×10^5/s for the decay rate and 1.28×10^9 y = 4.04×10^{16} s for the half-life.

50. (a) The disintegration energy for uranium-235 "decaying" into thorium-232 is

$$\begin{aligned} Q_3 &= \left(m_{^{235}\text{U}} - m_{^{232}\text{Th}} - m_{^{3}\text{He}}\right)c^2 = (235.0439\,\text{u} - 232.0381\,\text{u} - 3.0160\,\text{u})(931.5\,\text{MeV/u}) \\ &= -9.50\,\text{MeV}. \end{aligned}$$

(b) Similarly, the disintegration energy for uranium-235 decaying into thorium-231 is

$$\begin{aligned} Q_4 &= \left(m_{^{235}\text{U}} - m_{^{231}\text{Th}} - m_{^{4}\text{He}}\right)c^2 = (235.0439\,\text{u} - 231.0363\,\text{u} - 4.0026\,\text{u})(931.5\,\text{MeV/u}) \\ &= 4.66\,\text{MeV}. \end{aligned}$$

(c) Finally, the considered transmutation of uranium-235 into thorium-230 has a Q-value of

$$\begin{aligned} Q_5 &= \left(m_{^{235}\text{U}} - m_{^{230}\text{Th}} - m_{^{5}\text{He}}\right)c^2 = (235.0439\,\text{u} - 230.0331\,\text{u} - 5.0122\,\text{u})(931.5\,\text{MeV/u}) \\ &= -1.30\,\text{MeV}. \end{aligned}$$

Only the second decay process (the α decay) is spontaneous, as it releases energy.

57. (a) Since the positron has the same mass as an electron, and the neutrino has negligible mass, then

$$\Delta m c^2 = \left(\mathbf{m}_{\text{B}} + m_e - \mathbf{m}_{\text{C}}\right)c^2.$$

Now, since Carbon has 6 electrons (see Appendix F and/or G) and Boron has 5 electrons, we can add and subtract $6m_e$ to the above expression and obtain

$$\Delta m c^2 = \left(\mathbf{m}_{\text{B}} + 7m_e - \mathbf{m}_{\text{C}} - 6m_e\right)c^2 = \left(m_{\text{B}} + 2m_e - m_{\text{C}}\right)c^2.$$

We note that our final expression for Δmc^2 involves the *atomic* masses, as well an "extra" term corresponding to two electron masses. From Eq. 37-50 and Table 37-3, we obtain

$$Q = \left(m_{\text{C}} - m_{\text{B}} - 2m_e\right)c^2 = \left(m_{\text{C}} - m_{\text{B}}\right)c^2 - 2(0.511\,\text{MeV}).$$

(b) The disintegration energy for the positron decay of Carbon-11 is

$$Q = (11.011434\,\text{u} - 11.009305\,\text{u})(931.5\,\text{MeV/u}) - 1.022\,\text{MeV}$$
$$= 0.961\,\text{MeV}.$$

60. We solve for t from $R = R_0 e^{-\lambda t}$:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \left(\frac{5730\,\text{y}}{\ln 2}\right)\ln\left[\left(\frac{15.3}{63.0}\right)\left(\frac{5.00}{1.00}\right)\right] = 1.61\times 10^3\ \text{y}.$$

79. Since the spreading is assumed uniform, the count rate R = 74,000/s is given by

$$R = \lambda N = \lambda(M/m)(a/A),$$

where M = 400 g, m is the mass of the ^{90}Sr nucleus, A = 2000 km^2, and a is the area in question. We solve for a:

$$a = A\left(\frac{m}{M}\right)\left(\frac{R}{\lambda}\right) = \frac{AmRT_{1/2}}{M\ln 2}$$
$$= \frac{(2000\times 10^6\ \text{m}^2)(90\ \text{g/mol})(29\ \text{y})(3.15\times 10^7\ \text{s/y})(74{,}000/\text{s})}{(400\ \text{g})(6.02\times 10^{23}/\text{mol})(\ln 2)}$$
$$= 7.3\times 10^{-2}\ \text{m}^{-2} = 730\ \text{cm}^2.$$