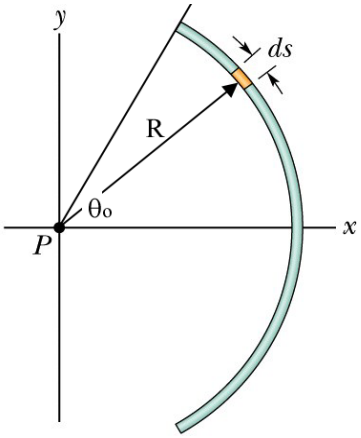

Sample Problem: Suppose we have a plastic rod bent into the shape of an arc, as shown at left. The opening angle of the rod is θ_o, as measured from the x-axis. The total charge on the rod is Q, and the radius of the arc is R. What is the net electric field at the center of the rod (point P)?

Solution: We start with the basic equation, $dE = (k\, dq / r^2)\, \boldsymbol{r}$. Our problem, just from a mathematics viewpoint, is that we need to convert this into a single-variable integral. In this case it is reasonably obvious that our variable must be θ, so we will work towards converting everything to that.

The unit vector $\boldsymbol{r}$ can be written as $\cos\theta\, \boldsymbol{i} + \sin\theta\, \boldsymbol{j}$. By inspection of the illustration, we can see that every point *ds* on the top of the arc will have an equal partner on the bottom of the arc. Thus, their y-components will always cancel, so the integral across $\boldsymbol{j}$ must be zero. The $\cos\theta\, \boldsymbol{i}$ term is fine as is, since θ is our chosen variable for the integral.

For dq, we have $dq = \lambda\, ds$, where $\lambda = Q/2\theta_o R$ (linear charge density) and $ds = R\, d\theta$. So, $dq = (Q/2\theta_o)\, d\theta$. In this case $r = R$, so that finishes setting up our integral: $dE = k\, (Q/2\theta_o R^2) \cos\theta\, d\theta$.

The integral is just sinθ, which we can either evaluate from $-\theta_o$ to $+\theta_o$, or from 0 to θ_o and double. In either case we have: $\mathbf{E} = (kQ/R^2)(\sin\theta_o / \theta_o)\, \boldsymbol{i}$.