Phyx 135-1 Final Exam, Spring 2017 **Name: ______________________________**

1) The mass of the planet Venus is 4.8685×10^{24} kg. It has a radius of 6052 km. Suppose you place a satellite with a mass of 5000 kg into orbit at a distance of 400 km above the surface of Venus.

1a) (4 points) What would be the velocity of this satellite?

Solution
We know $GMm/r^2 = mv^2/r$, because the gravitational force must equal the centrifugal force acting on the satellite. This gives us $v = (GM/r)^{1/2} = [(6.673 \times 10^{-11})(4.8685 \times 10^{24})/(6.452 \times 10^6)]^{1/2}$ = 7096 m/s.

1b) (6 points) You now ignite a booster rocket, and instantly increase the kinetic energy of the satellite by 3.4×10^{10} J. The satellite then moves into a new circular orbit. How far above the surface of Venus is this new orbit? (Hint: think about the *total* energy of the satellite, not just the kinetic.)

Solution
The total energy of the satellite is always $E = U + K = -GMm/r + ½ mv^2 = -GMm/2r$. Therefore, before it was given a boost, it had a total energy of $-(6.673 \times 10^{-11})(4.8685 \times 10^{24})(5000)/2(6.452 \times 10^6) = -1.259 \times 10^{11}$ J. Adding 3.4×10^{10} J of kinetic energy gives it a new total energy of -9.188×10^{10} J. After it moves into a new orbit, this new total energy will correspond to a new orbital radius of $-9.188 \times 10^{10} = -GMm/2r$, or $r = (6.673 \times 10^{-11})(4.8685 \times 10^{24})(5000)/2(9.188 \times 10^{10})$ = 8840 km. The new orbit is 8840 – 6052 = 2788 km above the surface of Venus.

2) (10 points) In a harbor in Lake Michigan there is a flat-bottomed barge which is anchored at a pier. The barge is 180 m long and 40 m wide. The sides of the barge (as measured from the flat bottom of the boat) are 15 m high. The unloaded barge has a mass of ten million kg. It is being loaded with a particular mineral ore which has a specific density of 3.1 g/cm^3. How many cubic meters of the mineral ore can be loaded onto the barge before the lake water comes over the side of the barge and sinks it?

Solution

The maximum volume of water that the barge can displace is 180 x 40 x 15 = 108,000 m^3 before it will start to sink. This corresponds to $m = \rho V = (10^3 \text{ kg/m}^3)(108,000 \text{ m}^3) = 1.08 \times 10^8$ kg. Therefore, the maximum mass of the loaded barge is also 1.08×10^8 kg, which means that the maximum mass of the mineral ore is $1.08 \times 10^8 - 10^7 = 9.8 \times 10^7$ kg. Since the ore has a density of 3.1 g/cm^3 = 3100 kg/m^3, the ship can hold $(9.8 \times 10^7)/(3100)$ = 31,613 m^3 of ore.

3) (10 points) An old-fashioned grandfather's clock has a pendulum which consists of a uniform bar that is 1.5 meters long and has a mass of 5 kg. Attached to the lower end of the bar (with its center exactly at the end of the bar) is a disk with a radius of 10 cm and a mass of 8 kg. If the pivot for the whole pendulum is at the very top of the bar, what is the period of the pendulum's swing? (Hint: you will need to break this problem into several pieces. To gain partial credit, it would be wise to explain what you are doing and why.)

Solution

We know that the angular frequency of a physical pendulum is $\omega = [mgh / I)]^{1/2}$. The problem in this case is figuring out h and I. Let us start with h, which is the distance to the CM of the pendulum.

The bar has its CM at 1.5 / 2 = 0.75 m from the top, and the disk has its CM at 1.5 m from the top. Therefore the CM of the pendulum is [(0.75)(5) + (1.5)(8)] / (5 + 8) = 1.21 m from the top.

The moment of inertia of the bar around one end is just $^1/_3\ mL^2 = (^1/_3)(5)(1.5)^2 = 3.75$ kg m^2.

The moment of inertia of the disk around the pivot will be $I = md^2 + I_{cm} = mL^2 + ½\ mR^2 =$ $(8)(1.5)^2 + ½\ (8)(0.1)^2 = 18.04$

So, the I of the whole pendulum is 3.75 + 18.04 = 21.79 kg m^2.

Inserting numbers into our formula for ω gives us: $\omega^2 = (13)(9.8)(1.21) / (21.79)$, or $\omega = 2.66$ rad/s. This corresponds to $T = 2\pi/\omega =$ 2.36 s.

4) (10 points) Let us assume that the average person has lungs which measure 30 cm by 25 cm in terms of the area they occupy on the person's chest. Let us also assume that the average person can no longer breathe if a weight of more than 150 lbs = 668 N is placed upon the front of their chest. Now, let us assume that this person is completely submerged in water, but is trying to breathe through a tube that extends up to the surface of the water. How deeply can the person be submerged before he can no longer successfully breathe?

Solution

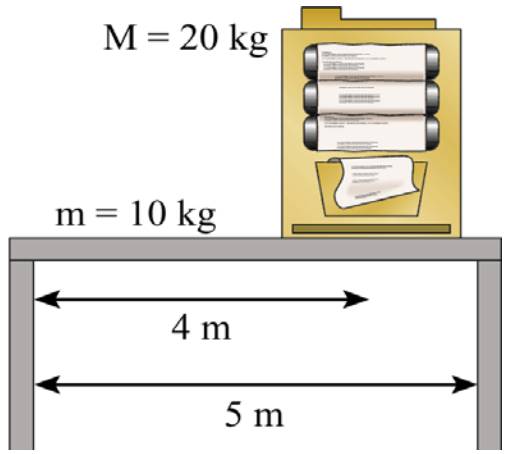
From the data given, we see that the person will no longer be able to breathe if a pressure of $P = F/A = 668 / (0.3 \times 0.25) = 8907$ Pa is placed on their chest. This pressure represents an *excess* pressure, of course, since the person already has atmospheric pressure inside their lungs. The formula $P = P_o + \rho gh$ gives us the absolute pressure in the water as a function of depth. However, the P_o cancels out because there is atmospheric pressure on top of the water, but also atmospheric pressure inside the person's lungs. We therefore have $8907 = \rho gh$, or $h = 8907 / (10^3)(9.8) =$ 0.91 m.

5) (10 points) A thin uniform bar has a pivot fixed at one end, and a spring fixed to the other end such that it is at 90° to the length of the bar. The bar has a length of L = 1.5 m, and a mass of m = 0.75 kg. The spring has a spring constant of k = 1200 N/m. If the bar is nudged slightly, and set into oscillating motion, what will be the period of its oscillations?

Solution

First we must write down an equation of motion for the bar. Using $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, we have $\tau = -Lkx$, where x is the slight amount that the bar is nudged. But, we also have $\tau = I\alpha$, which in this case corresponds to $\tau = (^1/_3)mL^2\, d^2\theta/dt^2$. Using the fact that $x = L\theta$ if x is small, we can then equate the two formulas for τ to get: $-L^2k\theta = (^1/_3)mL^2\, d^2\theta/dt^2$. We immediately recognize this as the formula for SHM, which means that it has $\omega = [L^2k / (^1/_3)mL^2]^{1/2} = (3k/m)^{1/2} = (3600/0.75)^{1/2} = 69.3$ rad/s. This corresponds to a period of $T = 2\pi/\omega = 0.091$ s.

6) (10 points) A table which is 5 m long and has a mass of m = 10 kg is holding up a printer which has a mass of M = 20 kg. The center of mass of the printer is located 4 m from the left end of the table. What is the force in newtons pressing down on the left and right ends of the table?



Solution

We know that the total force pressing down on both ends of the table must equal the total weight of the objects, so we have: $F_L + F_R = (m + M)g = (30)(9.8) = 294$ N.

To get a second equation for F_R, we can place an imaginary pivot directly above the left side of the table and then compute the torques about that point. We know that the total torque must equal zero (because the system is not moving), so we have:

$$\tau = rF = -(2.5\text{ m})(10\text{ kg})(9.8\text{ m/s}^2) - (4\text{ m})(20\text{ kg})(9.8\text{ m/s}^2) + (5\text{ m})F_R = 0$$

This yields F_R = 206 N. Therefore $F_L = 294 - 206 =$ 88 N.

7) (10 points) Two hockey pucks are sliding across frictionless ice. The first puck has a mass of $m_1 = 0.5$ kg and is moving at $v_1 = 5$ m/s directly along the positive x-axis. The second puck has a mass of $m_2 = 1.0$ kg and is moving at $v_2 = 8$ m/s directly along the positive y-axis. Then they collide and stick together. At what angle (measured from the positive x-axis) will the two pucks continue their motion?

Solution

The "after" velocity components of the two pucks can be calculated using conservation of momentum.

For the x-direction, we have $(0.5)(5) = (1.5)v_x$, or $v_x = 1.667$ m/s.
For the y-direction, we have $(1.0)(8) = (1.5)v_y$, or $v_y = 5.333$ m/s.

The angle of their motion is given by $\tan\theta = 5.333 / 1.667$, or $\theta = 72.6°$

Multiple Choice (2 points each). Select the one best answer.

_____ **8)** The Earth does not remain stationary as the Moon orbits it, but instead orbits an axis that represents the center-of-mass of the Earth-Moon system. This axis passes through the Earth at a point $^2/_3$ of the way outward from the center of the Earth. What is the moment of inertia of the Earth about this axis? You may assume that the Earth is a perfect sphere. (M = mass of Earth, R = radius of Earth).

A) 0.13 MR^2	B) 0.84 MR^2	C) 1.5 MR^2	D) 0.70 MR^2
E) 0.25 MR^2	F) 1.1 MR^2	G) 0.47 MR^2	H) 1.7 MR^2

Answer: From the parallel axis theorem, we have: $I = Mh^2 + I_{cm} = M(2R/3)^2 + {}^2/_5\,MR^2 = 0.84\ MR^2$

_____ **9)** Suppose a spaceship is taking off from Mars with an acceleration of 4.5 m/s^2 relative to the Martian surface. (The surface gravity on Mars is 38% that of Earth.) A fish setting on a scale on board the spaceship is causing the scale to register 28 N. What is the mass of the fish?

A) 6.2 kg	B) 35 kg	C) 7.6 kg	D) 3.4 kg
E) 2.4 kg	F) 1.1 kg	G) 0.47 kg	H) 1.7 kg

Answer: The acceleration acting on the fish is 4.5 m/s^2 + (0.38)(9.8 m/s^2) = 8.2 m/s^2. Thus the mass of the fish is m = F/a = 28/8.2 = 3.4 kg.

_____ **10)** You are standing on a cart which has perfectly frictionless wheels. The cart is not moving. You have a mass of 80 kg, and the cart has a mass of 40 kg. Then, you toss an 8-kg cannonball at 5 m/s directly along the length of the cart. After you toss the cannonball, the speed of the center of mass of the cart + you + cannonball will be:

A) 0.33 m/s	B) 0.50 m/s	C) 1.0 m/s	D) 5.0 m/s
E) 0.25 m/s	F) 1.2 m/s	G) zero	H) 8.0 m/s

Answer: The center of mass of any system cannot move unless there is an *outside* force operating on it. Since you and the cannonball are part of the system, the CM speed will be zero.

_____ **11)** An iceskater with a moment of inertia of I = 12 kg m^2 is rotating at 75 rpm. Then, she pulls in her arms and begins rotating at 100 rpm. What is her moment of inertia after pulling in her arms?

A) 12 kg m^2	B) 21 kg m^2	C) 16 kg m^2	D) 14 kg m^2
E) 18 kg m^2	F) 9 kg m^2	G) 6 kg m^2	H) 3 kg m^2

Answer: Her angular momentum $L = I\omega$ will not change as she moves her arms, so we have $(12)(75) = I_2(100)$, or I_2 = 9 kg m^2.

_____ **12)** A ball of mass = 0.5 kg and radius = 3 cm is setting at the top of a ramp which is 1.5 m in height. Then, the ball rolls to the bottom of the ramp. How fast will the center of mass of the ball be moving?

A) 5.42 m/s	B) 5.22 m/s	C) 4.91 m/s	D) 4.77 m/s
E) 4.58 m/s	F) 4.38 m/s	G) 3.93 m/s	H) 3.80 m/s

Solution: The ball's kinetic energy will be $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$. However, since $v = R\omega$ for a rolling object, we see that $E = \frac{1}{2} mv^2 + \frac{1}{2} (^2/_5)mR^2(v/R)^2 = \frac{1}{2} mv^2 + {}^1/_5\, mv^2 = 0.7\, mv^2$. This will equal to the gravitational potential energy, or mgh. We have $gh = 0.7\, v^2$, or v = 4.58 m/s.

Some Moments of Inertia

Uniform bar around its center: $^{1}/_{12}$ ML^2
Uniform bar around one end: $^{1}/_{3}$ ML^2
Uniform disk around its center: ½ MR^2
Uniform sphere around its center: $^{2}/_{5}$ MR^2