3. We use Eq. 9-5 to locate the coordinates.

(a) By symmetry $x_{com} = -d_1/2 = -(13\text{ cm})/2 = -6.5$ cm. The negative value is due to our choice of the origin.

(b) We find y_{com} as

$$y_{com} = \frac{m_i y_{com,i} + m_a y_{com,a}}{m_i + m_a} = \frac{\rho_i V_i y_{com,i} + \rho_a V_a y_{cm,a}}{\rho_i V_i + \rho_a V_a}$$

$$= \frac{(11\text{ cm}/2)(7.85\text{ g/cm}^3) + 3(11\text{ cm}/2)(2.7\text{ g/cm}^3)}{7.85\text{ g/cm}^3 + 2.7\text{ g/cm}^3} = 8.3\text{ cm.}$$

(c) Again by symmetry, we have $z_{com} = (2.8\text{ cm})/2 = 1.4$ cm.

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the left end of the tabletop (as shown in Fig. 9-37). With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x,y) = (+L, -L/2)$, the center of mass of the left leg is at $(x,y) = (0, -L/2)$, and the center of mass of the tabletop is at $(x,y) = (L/2, 0)$.

(a) The x coordinate of the (whole table) center of mass is

$$x_{com} = \frac{M(+L) + M(0) + 3M(+L/2)}{M + M + 3M} = \frac{L}{2}.$$

With $L = 22$ cm, we have $x_{com} = (22\text{ cm})/2 = 11$ cm.

(b) The y coordinate of the (whole table) center of mass is

$$y_{com} = \frac{M(-L/2) + M(-L/2) + 3M(0)}{M + M + 3M} = -\frac{L}{5},$$

or $y_{com} = -(22\text{ cm})/5 = -4.4$ cm.

From the coordinates, we see that the whole table center of mass is a small distance 4.4 cm directly below the middle of the tabletop.

13. **THINK** A shell explodes into two segments at the top of its trajectory. Knowing the motion of one segment allows us to analyze the motion of the other using the momentum conservation principle.

EXPRESS We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60°\cos 60° = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60° = 15.3\text{ m}.$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. In addition, since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then

$$Mv_0\cos\theta_0 = MV_0/2,$$

since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60° = 20\text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands.

ANALYZE Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m}.$$

LEARN In the absence of explosion, the shell with a mass M would have landed at

$$R = 2x_0 = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin[2(60°)] = 35.3\text{ m}$$

which is shorter than $x = 53$ m found above. This makes sense because the broken fragment, having a smaller mass but greater horizontal speed, can travel much farther than the original shell.

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16,

$$M\Delta x_{\text{com}} = 0 = m_b \Delta x_b + m_d \Delta x_d ,$$

which implies

$$\left|\Delta x_b\right| = \frac{m_d}{m_b}\left|\Delta x_d\right|.$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance d = 2.4 m:

$$\left|\Delta x_b\right| + \left|\Delta x_d\right| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}\left|\left(\Delta x_d\right)\right| + \left|\Delta x_d\right| = d$$

which leads to $\left|\Delta x_d\right| = \dfrac{d}{1 + m_d / m_b} = \dfrac{2.4\text{ m}}{1 + (4.5/18)} = 1.92\text{ m}.$

The dog is therefore 1.9 m closer to the shore than initially (where it was D = 6.1 m from it). Thus, it is now $D - |\Delta x_d|$ = 4.2 m from the shore.

21. We use coordinates with $+x$ horizontally toward the pitcher and $+y$ upward. Angles are measured counterclockwise from the $+x$ axis. Mass, velocity, and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = \left(4.5 \angle 215°\right)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is

$$(6.0 \angle -90°) - (4.5 \angle 215°) = (5.0 \angle -43°)$$

(efficiently done with a vector-capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 kg·m/s.

(b) The momentum change is $(6.0 \angle 0°) - (4.5 \angle 215°) = (10 \angle 15°)$. Thus, the magnitude of the momentum change is 10 kg·m/s.