

6. If we make the units explicit, the function is

$$\theta = (4.0 \text{ rad/s})t - (3.0 \text{ rad/s}^2)t^2 + (1.0 \text{ rad/s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Equation 10-6 leads to

$$\omega = \frac{d}{dt}(4t - 3t^2 + t^3) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(4 - 6t + 3t^2) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s².

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s². We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

13. The wheel has angular velocity $\omega_0 = +1.5$ rad/s = +0.239 rev/s at $t = 0$, and has constant value of angular acceleration $\alpha < 0$, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = +20$ rev, and at t_2 its angular displacement is $\theta_2 = +40$ rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 10-15:

$$\theta_2 = \frac{1}{2}(\omega_0 + \omega_2)t_2 \Rightarrow t_2 = \frac{2(40 \text{ rev})}{0.239 \text{ rev/s}} = 335 \text{ s}$$

which we round off to $t_2 \approx 3.4 \times 10^2$ s.

(b) Any equation in Table 10-1 involving α can be used to find the angular acceleration; we select Eq. 10-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2} \alpha t_2^2 \Rightarrow \alpha = -\frac{2(40 \text{ rev})}{(335 \text{ s})^2} = -7.12 \times 10^{-4} \text{ rev/s}^2$$

which we convert to $\alpha = -4.5 \times 10^{-3} \text{ rad/s}^2$.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2} \alpha t_1^2$ (Eq. 10-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1 \alpha}}{\alpha} = \frac{-(0.239 \text{ rev/s}) \pm \sqrt{(0.239 \text{ rev/s})^2 + 2(20 \text{ rev})(-7.12 \times 10^{-4} \text{ rev/s}^2)}}{-7.12 \times 10^{-4} \text{ rev/s}^2}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is $t_1 = 98$ s.

18. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2} \frac{dT}{dt}.$$

For the pulsar described in the problem, we have

$$\frac{dT}{dt} = \frac{1.26 \times 10^{-5} \text{ s/y}}{3.16 \times 10^7 \text{ s/y}} = 4.00 \times 10^{-13}.$$

Therefore,

$$\alpha = -\left(\frac{2\pi}{(0.033 \text{ s})^2}\right)(4.00 \times 10^{-13}) = -2.3 \times 10^{-9} \text{ rad/s}^2.$$

The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

(b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:

$$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3 \times 10^{-9} \text{ rad/s}^2)(0.033 \text{ s})} = 8.3 \times 10^{10} \text{ s} \approx 2.6 \times 10^3 \text{ years}$$

(c) The pulsar was born $1992 - 1054 = 938$ years ago. This is equivalent to $(938 \text{ y})(3.16 \times 10^7 \text{ s/y}) = 2.96 \times 10^{10} \text{ s}$. Its angular velocity at that time was

$$\omega = \omega_0 + \alpha t + \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033 \text{ s}} + (-2.3 \times 10^{-9} \text{ rad/s}^2)(-2.96 \times 10^{10} \text{ s}) = 258 \text{ rad/s}.$$

Its period was

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{258 \text{ rad/s}} = 2.4 \times 10^{-2} \text{ s}.$$

27. (a) The angular speed in rad/s is

$$\omega = \left(33 \frac{1}{3} \text{ rev/min} \right) \left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}} \right) = 3.49 \text{ rad/s}.$$

Consequently, the radial (centripetal) acceleration is (using Eq. 10-23)

$$a = \omega^2 r = (3.49 \text{ rad/s})^2 (6.0 \times 10^{-2} \text{ m}) = 0.73 \text{ m/s}^2.$$

(b) Using Ch. 6 methods, we have $ma = f_s \leq f_{s,\max} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:

$$\mu_{s,\min} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075.$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus,

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2}.$$

If the object is not to slip at any time, we require

$$f_{s,\max} = \mu_s mg = ma_{\max} = mr\sqrt{\omega_{\max}^4 + \alpha^2}.$$

Thus, since $\alpha = \omega/t$ (from Eq. 10-12), we find

$$\mu_{s,\min} = \frac{r\sqrt{\omega_{\max}^4 + \alpha^2}}{g} = \frac{r\sqrt{\omega_{\max}^4 + (\omega_{\max}/t)^2}}{g} = \frac{(0.060)\sqrt{3.49^4 + (3.4/0.25)^2}}{9.8} = 0.11.$$

29. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}$ rad. That time is

$$t = \frac{2\ell}{c} = \frac{2(500 \text{ m})}{2.998 \times 10^8 \text{ m/s}} = 3.34 \times 10^{-6} \text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26 \times 10^{-2} \text{ rad}}{3.34 \times 10^{-6} \text{ s}} = 3.8 \times 10^3 \text{ rad/s.}$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = (3.8 \times 10^3 \text{ rad/s})(0.050 \text{ m}) = 1.9 \times 10^2 \text{ m/s.}$$