7. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30^\circ = 3.0$ m (we are using the edge of the roof as our reference level for computing *U*). Its final kinetic energy (as it leaves the roof) is (Eq. 11-5)

$$K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2.$$

Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where R = 0.10 m. Using $I = \frac{1}{2}MR^2$ (Table 10-2(c)), conservation of energy leads to

$$Mgh = \frac{1}{2}Mv^{2} + \frac{1}{2}I\omega^{2} = \frac{1}{2}MR^{2}\omega^{2} + \frac{1}{4}MR^{2}\omega^{2} = \frac{3}{4}MR^{2}\omega^{2}.$$

The mass *M* cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10 \text{ m}}\sqrt{\frac{4}{3}(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 63 \text{ rad/s}.$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take +x leftward and +y downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$v_{0x} = v_0 \cos 30^\circ = 5.4 \text{ m/s}$$

 $v_{0y} = v_0 \sin 30^\circ = 3.1 \text{ m/s}.$

The projectile motion equations become

$$x = v_{0x}t$$
 and $y = v_{0y}t + \frac{1}{2}gt^2$.

We first find the time when y = H = 5.0 m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74 \,\mathrm{s}.$$

Then we substitute this into the x equation and obtain x = (5.4 m/s)(0.74 s) = 4.0 m.

10. From $I = \frac{2}{3}MR^2$ (Table 10-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3(0.040 \text{ kg} \cdot \text{m}^2)}{2(0.15 \text{ m})^2} = 2.7 \text{ kg}.$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{com} = R\omega$, and we find

$$\frac{K_{\rm rot}}{K_{\rm com} + K_{\rm rot}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2}.$$

(a) Simplifying the above ratio, we find $K_{rot}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or

$$K_{\rm rot} = (0.4)(20 \text{ J}) = 8.0 \text{ J}.$$

(b) From $K_{\text{rot}} = \frac{1}{3}M R^2 \omega^2 = 8.0 \text{ J}$ (and using the above result for *M*) we find

$$\omega = \frac{1}{0.15 \text{ m}} \sqrt{\frac{3(8.0 \text{ J})}{2.7 \text{ kg}}} = 20 \text{ rad/s}$$

which leads to $v_{\text{com}} = (0.15 \text{ m})(20 \text{ rad/s}) = 3.0 \text{ m/s}.$

(c) We note that the inclined distance of 1.0 m corresponds to a height $h = 1.0 \sin 30^\circ = 0.50$ m. Mechanical energy conservation leads to

$$K_i = K_f + U_f \implies 20 \,\mathrm{J} = K_f + Mgh$$

which yields (using the values of *M* and *h* found above) $K_f = 6.9$ J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \implies \omega_f = \frac{1}{0.15 \text{ m}}\sqrt{\frac{3(0.40)(6.9 \text{ J})}{2.7 \text{ kg}}}$$

which yields $\omega_f = 12$ rad/s and leads to

 $v_{\text{com}f} = R\omega_f = (0.15 \text{ m})(12 \text{ rad/s}) = 1.8 \text{ m/s}.$

11. With $\vec{F}_{app} = (10 \text{ N})\hat{i}$, we solve the problem by applying Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F_{\rm app} - f_s = ma \implies f_s = 10 \,\mathrm{N} - (10 \,\mathrm{kg}) (0.60 \,\mathrm{m/s^2}) = 4.0 \,\mathrm{N}.$$

In unit vector notation, we have $\vec{f}_s = (-4.0 \text{ N})\hat{i}$, which points leftward.

(b) With R = 0.30 m, we find the magnitude of the angular acceleration to be

$$|\alpha| = |a_{\rm com}| / R = 2.0 \text{ rad/s}^2,$$

from Eq. 11-6. The only force not directed toward (or away from) the center of mass is \vec{f}_s , and the torque it produces is clockwise:

$$|\tau| = I |\alpha| \implies (0.30 \,\mathrm{m})(4.0 \,\mathrm{N}) = I (2.0 \,\mathrm{rad/s^2})$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60 \text{ kg} \cdot \text{m}^2$.

18. (a) The derivation of the acceleration is found in § 11-4; Eq. 11-13 gives

$$a_{\rm com} = -\frac{g}{1 + I_{\rm com}/MR_0^2}$$

where the positive direction is upward. We use $I_{com} = MR^2/2$ where the radius is R = 0.32 m and M = 116 kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + (MR^2/2)/MR_0^2} = \frac{g}{1 + (R/R_0)^2/2}$$

which yields a = -g/51 upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s².

(b) As observed in §11-4, our result in part (a) applies to both the descending and the rising yo-yo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \Rightarrow T = M\left(g - \frac{g}{51}\right) = 1.1 \times 10^3 \text{ N}.$$

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yo-yo will involve a larger cord tension.