23. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\mathrm{i}} + \left(z'F_x - x'F_z\right)\hat{\mathrm{j}} + \left(x'F_y - y'F_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) x = 0, y = 0.5, z = –2.0, F_x = 2.0, F_y = 0, and F_z = –3.0. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}} - 1.0\hat{\mathrm{k}}\right)\,\mathrm{N \cdot m}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = -2.0, y' = 0.5, z' = 1.0, F_x = 2.0, F_y = 0$, and $F_z = -3.0.$ Thus, we obtain

$$\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} - 1.0\hat{\mathrm{k}}\right)\ \mathrm{N \cdot m}.$$

30. (a) The acceleration vector is obtained by dividing the force vector by the (scalar) mass:

$$\vec{a} = \vec{F}/m = (3.00\ \mathrm{m/s^2})\hat{\mathrm{i}} - (4.00\ \mathrm{m/s^2})\hat{\mathrm{j}} + (2.00\ \mathrm{m/s^2})\hat{\mathrm{k}}.$$

(b) Use of Eq. 11-18 leads directly to

$$\vec{L} = (42.0\ \mathrm{kg{\cdot}m^2/s})\hat{\mathrm{i}} + (24.0\ \mathrm{kg{\cdot}m^2/s})\hat{\mathrm{j}} + (60.0\ \mathrm{kg{\cdot}m^2/s})\hat{\mathrm{k}}.$$

(c) Similarly, the torque is

$$\vec{\tau} = \vec{r} \times \vec{F} = (-8.00\ \mathrm{N \cdot m})\hat{\mathrm{i}} - (26.0\ \mathrm{N \cdot m})\hat{\mathrm{j}} - (40.0\ \mathrm{N \cdot m})\hat{\mathrm{k}}.$$

(d) We note (using the Pythagorean theorem) that the magnitude of the velocity vector is 7.35 m/s and that of the force is 10.8 N. The dot product of these two vectors is $\vec{\mathrm{v}} \cdot \vec{F} = -48$ (in SI units). Thus, Eq. 3-20 yields

$$\theta = \cos^{-1}[-48.0/(7.35 \times 10.8)] = 127°.$$

38. (a) Equation 10-34 gives $\alpha = \tau/I$ and Eq. 10-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at t = 0.033 s is

$$I\omega = \tau\, t = \left(16\,\mathrm{N \cdot m}\right)\left(0.033\mathrm{s}\right) = 0.53\,\mathrm{kg \cdot m^2/s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau\, t}{I} = \frac{(16\ \mathrm{N\cdot m})(0.033\,\mathrm{s})}{1.2\times 10^{-3}\ \mathrm{kg\cdot m^2}} = 440\,\mathrm{rad/s}$$

which we convert as follows:

$$\omega = (440\ \mathrm{rad/s})(60\ \mathrm{s/min})(1\ \mathrm{rev}/2\pi\ \mathrm{rad}) \approx 4.2\times 10^{3}\ \mathrm{rev/min}.$$

44. So that we don't get confused about ± signs, we write the angular *speed* to the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I|\omega_0| = -\left(mR^2 + I\right)|\omega_f|.$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I|\omega_0|}{mR^2 + I} = \frac{(0.17\ \mathrm{kg})(2.0\ \mathrm{m/s})(0.15\ \mathrm{m}) - (5.0\times 10^{-3}\ \mathrm{kg\cdot m^2})(2.8\ \mathrm{rad/s})}{(5.0\times 10^{-3}\ \mathrm{kg\cdot m^2}) + (0.17\ \mathrm{kg})(0.15\ \mathrm{m})^2}$$
$$= 4.2\ \mathrm{rad/s}.$$

(b) No, $K_f \neq K_i$ and — if desired — we can solve for the difference:

$$K_i - K_f = \frac{mI}{2}\,\frac{v^2 + \omega_0^2 R^2 + 2Rv|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" — that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

53. The axis of rotation is in the middle of the rod, with r = 0.25 m from either end. By Eq. 11-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\theta$ where m = 0.003 kg and θ = 60°. Relative to the axis, this is

counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is

$$I = I_{rod} + mr^2$$

where $I_{rod} = ML^2/12$ by Table 10-2(e), with M = 4.0 kg and L = 0.5 m. Angular momentum conservation leads to

$$rmv\sin\theta = \left(\frac{1}{12}ML^2 + mr^2\right)\omega.$$

Thus, with ω = 10 rad/s, we obtain

$$v = \frac{\left(\frac{1}{12}(4.0\text{ kg})(0.5\text{ m})^2 + (0.003\text{ kg})(0.25\text{ m})^2\right)(10\text{ rad/s})}{(0.25\text{ m})(0.003\text{ kg})\sin 60°} = 1.3\times10^3\text{ m/s}.$$