45. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4\times 10^6 \text{ m})^3}{(6.67\times 10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})\left(2.754\times 10^4 \text{ s}\right)^2} = 6.5\times 10^{23} \text{ kg}.$$

51. **THINK** The satellite moves in an elliptical orbit about Earth. An elliptical orbit can be characterized by its semi-major axis and eccentricity.

EXPRESS The greatest distance between the satellite and Earth's center (the apogee distance) and the least distance (perigee distance) are, respectively,

$$R_a = R_E + d_a = 6.37 \times 10^6 \text{ m} + 360 \times 10^3 \text{ m} = 6.73 \times 10^6 \text{ m}$$
$$R_p = R_E + d_p = 6.37 \times 10^6 \text{ m} + 180 \times 10^3 \text{ m} = 6.55 \times 10^6 \text{ m}.$$

Here $R_E = 6.37 \times 10^6$ m is the radius of Earth.

ANALYZE The semi-major axis is given by

$$a = \frac{R_a + R_p}{2} = \frac{6.73\times 10^6 \text{ m} + 6.55\times 10^6 \text{ m}}{2} = 6.64\times 10^6 \text{ m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73\times 10^6 \text{ m} - 6.55\times 10^6 \text{ m}}{6.73\times 10^6 \text{ m} + 6.55\times 10^6 \text{ m}} = 0.0136.$$

LEARN Since e is very small, the orbit is nearly circular. On the other hand, if e is close to unity, then the orbit would be a long, thin ellipse.

54. The two stars are in circular orbits, not about each other, but about the two-star system's center of mass (denoted as O), which lies along the line connecting the centers of the two stars. The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, for the visible, Newton's second law gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r_1}$$

where r is the distance between the centers of the stars. To find the relation between r and r_1, we locate the center of mass relative to m_1. Using Equation 9-1, we obtain

$$r_1 = \frac{m_1(0) + m_2 r}{m_1 + m_2} = \frac{m_2 r}{m_1 + m_2} \quad \Rightarrow \quad r = \frac{m_1 + m_2}{m_2} r_1 .$$

On the other hand, since the orbital speed of m_1 is $v = 2\pi r_1 / T$, then $r_1 = vT / 2\pi$ and the expression for r can be rewritten as

$$r = \frac{m_1 + m_2}{m_2} \frac{vT}{2\pi} .$$

Substituting r and r_1 into the force equation, we obtain

$$F = \frac{4\pi^2 G m_1 m_2^3}{(m_1 + m_2)^2 v^2 T^2} = \frac{2\pi m_1 v}{T}$$

or

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3 T}{2\pi G} = \frac{(2.7\times 10^5 \text{ m/s})^3 (1.70 \text{ days})(86400 \text{ s/day})}{2\pi (6.67\times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)} = 6.90\times 10^{30} \text{ kg}$$
$$= 3.467 M_s ,$$

where $M_s = 1.99\times 10^{30}$ kg is the mass of the sun. With $m_1 = 6M_s$, we write $m_2 = \alpha M_s$ and solve the following cubic equation for α :

$$\frac{\alpha^3}{(6+\alpha)^2} - 3.467 = 0 .$$

The equation has one real solution: $\alpha = 9.3$, which implies $m_2 / M_s \approx 9$.

56. (a) The period is T = 27(3600) = 97200 s, and we are asked to assume that the orbit is circular (of radius r = 100000 m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)\left(100000\right)^3 \Rightarrow M = 6.3 \times 10^{16} \text{ kg}.$$

(b) Dividing the mass M by the given volume yields an average density equal to

$$\rho = (6.3 \times 10^{16} \text{ kg})/(1.41 \times 10^{13} \text{ m}^3) = 4.4 \times 10^3 \text{ kg/m}^3,$$

which is about 20% less dense than Earth.