

66. (a) The pellets will have the same speed v but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing v with $2v$ in Eq. 13-38 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4 \left(\frac{GM_E m}{2r} \right) = \frac{2(6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2) (5.98 \times 10^{24} \text{ kg})(0.0040 \text{ kg})}{(6370 + 500) \times 10^3 \text{ m}}$$

$$= 4.6 \times 10^5 \text{ J.}$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6 \times 10^5 \text{ J}}{\frac{1}{2}(0.0040 \text{ kg})(950 \text{ m/s})^2} = 2.6 \times 10^2.$$

67. (a) The force acting on the satellite has magnitude GMm/r^2 , where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r , where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite:

$$r = (6.37 \times 10^6 + 640 \times 10^3) \text{ m} = 7.01 \times 10^6 \text{ m.}$$

Thus,

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3 / \text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{7.01 \times 10^6 \text{ m}}} = 7.54 \times 10^3 \text{ m/s.}$$

(b) The period is

$$T = 2\pi r/v = 2\pi(7.01 \times 10^6 \text{ m})/(7.54 \times 10^3 \text{ m/s}) = 5.84 \times 10^3 \text{ s} \approx 97 \text{ min.}$$

(c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4 \times 10^5 \text{ J/orbit}$. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.

The initial energy is

$$E_0 = -\frac{(6.67 \times 10^{-11} \text{ m}^3 / \text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(220 \text{ kg})}{2(7.01 \times 10^6 \text{ m})} = -6.26 \times 10^9 \text{ J,}$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26 \times 10^9 \text{ J} - (1500 \text{ orbit})(1.4 \times 10^5 \text{ J/orbit}) = -6.47 \times 10^9 \text{ J,}$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(220 \text{ kg})}{2(-6.47 \times 10^9 \text{ J})} = 6.78 \times 10^6 \text{ m}.$$

The altitude is

$$h = r - R = (6.78 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}) = 4.1 \times 10^5 \text{ m}.$$

Here R is the radius of Earth. This torque is internal to the satellite–Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{6.78 \times 10^6 \text{ m}}} = 7.67 \times 10^3 \text{ m/s} \approx 7.7 \text{ km/s}.$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78 \times 10^6 \text{ m})}{7.67 \times 10^3 \text{ m/s}} = 5.6 \times 10^3 \text{ s} \approx 93 \text{ min}.$$

(f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01 \times 10^6 \text{ m}) = 4.40 \times 10^7 \text{ m}$, and $\Delta E = -1.4 \times 10^5 \text{ J}$. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4 \times 10^5 \text{ J}}{4.40 \times 10^7 \text{ m}} = 3.2 \times 10^{-3} \text{ N}.$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite–Earth system is essentially isolated, so its momentum is very nearly conserved.

68. The orbital radius is $r = R_E + h = 6370 \text{ km} + 400 \text{ km} = 6770 \text{ km} = 6.77 \times 10^6 \text{ m}$.

(a) Using Kepler's law given in Eq. 13-34, we find the period of the ships to be

$$T_0 = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (6.77 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}} = 5.54 \times 10^3 \text{ s} \approx 92.3 \text{ min}.$$

(b) The speed of the ships is

$$v_0 = \frac{2\pi r}{T_0} = \frac{2\pi(6.77 \times 10^6 \text{ m})}{5.54 \times 10^3 \text{ s}} = 7.68 \times 10^3 \text{ m/s}^2.$$

(c) The new kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(0.99v_0)^2 = \frac{1}{2}(2000 \text{ kg})(0.99)^2(7.68 \times 10^3 \text{ m/s})^2 = 5.78 \times 10^{10} \text{ J}.$$

(d) Immediately after the burst, the potential energy is the same as it was before the burst. Therefore,

$$U = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(2000 \text{ kg})}{6.77 \times 10^6 \text{ m}} = -1.18 \times 10^{11} \text{ J}.$$

(e) In the new elliptical orbit, the total energy is

$$E = K + U = 5.78 \times 10^{10} \text{ J} + (-1.18 \times 10^{11} \text{ J}) = -6.02 \times 10^{10} \text{ J}.$$

(f) For elliptical orbit, the total energy can be written as (see Eq. 13-42) $E = -GMm/2a$, where a is the semi-major axis. Thus,

$$a = -\frac{GMm}{2E} = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(2000 \text{ kg})}{2(-6.02 \times 10^{10} \text{ J})} = 6.63 \times 10^6 \text{ m}.$$

(g) To find the period, we use Eq. 13-34 but replace r with a . The result is

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2 (6.63 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}} = 5.37 \times 10^3 \text{ s} \approx 89.5 \text{ min}.$$

(h) The orbital period T for Picard's elliptical orbit is shorter than Igor's by

$$\Delta T = T_0 - T = 5540 \text{ s} - 5370 \text{ s} = 170 \text{ s}.$$

Thus, Picard will arrive back at point P ahead of Igor by $170 \text{ s} - 90 \text{ s} = 80 \text{ s}$.