5. **THINK** The blade of the shaver undergoes simple harmonic motion. We want to find its amplitude, maximum speed and maximum acceleration.

EXPRESS The amplitude x_m is half the range of the displacement D. Once the amplitude is known, the maximum speed v_m is related to the amplitude by $v_m = \omega x_m$, where ω is the angular frequency. Similarly, the maximum acceleration is $a_m = \omega^2 x_m$.

ANALYZE (a) The amplitude is $x_m = D/2 = (2.0 \text{ mm})/2 = 1.0 \text{ mm}$.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi (120 \text{ Hz})(1.0\times10^{-3} \text{ m}) = 0.75 \text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(120 \text{ Hz}))^2 (1.0\times10^{-3} \text{ m}) = 5.7\times10^2 \text{ m/s}^2.$$

LEARN In SHM, acceleration is proportional to the displacement x_m.

18. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x_m = \frac{1}{2}d$ and ω = 0.503 rad/h. The phase constant ϕ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$. With $x = x_m \cos(\omega t + \phi)$, we obtain

$$\frac{1}{4}d = \left(\frac{1}{2}d\right)\cos(0.503t + 0) \Rightarrow \frac{1}{2} = \cos(0.503t)$$

which has t = 2.08 h as the smallest positive root. The calculator is in radians mode during this calculation.

30. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where E = 1.00 J and x_m = 0.100 m. Thus,

$$k = \frac{2E}{x_m^2} = 200 \text{ N/m}.$$

(b) The energy as the block passes through the equilibrium position (with speed v_m = 1.20 m/s) is purely kinetic:

$$E=\frac{1}{2}mv_m^2 \quad \Rightarrow \quad m=\frac{2E}{v_m^2}=1.39 \text{ kg}.$$

(c) Equation 15-12 (divided by 2π) yields

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=1.91 \text{ Hz}.$$

38. From Eq. 15-23 (in absolute value) we find the torsion constant:

$$\kappa=\left|\frac{\tau}{\theta}\right|=\frac{0.20 \text{ N}\cdot\text{m}}{0.85 \text{ rad}}=0.235 \;\; \text{N}\cdot\text{m/rad}.$$

With $I = 2mR^2/5$ (the rotational inertia for a solid sphere — from Chapter 11), Eq. 15–23 leads to

$$T=2\pi\sqrt{\frac{\frac{2}{5}mR^2}{\kappa}}=2\pi\sqrt{\frac{\frac{2}{5}(95\text{ kg})(0.15\text{ m})^2}{0.235 \;\; \text{N}\cdot\text{m/rad}}}=12\text{ s}.$$

53. **THINK** By assuming that the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and that the torque tends to pull the rod toward its equilibrium orientation, we see that the rod will oscillate in simple harmonic motion.

EXPRESS Let $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega=\sqrt{C/I}$ and the period is

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I}{C}},$$

where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved a distance $(L/2)(1-\cos\theta)$ to the right. The length of the spring is now

$$\ell=\sqrt{(L/2)^2(1-\cos\theta)^2+[\ell_0+(L/2)\sin\theta]^2}.$$

If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus, $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$ (see Table 10-2), where m is its mass.

ANALYZE Substituting the expressions for C and I, we find the period of oscillation to be

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}.$$

With m = 0.600 kg and k = 1850 N/m, we obtain T = 0.0653 s.

LEARN As in the case of a simple linear harmonic oscillator formed by a mass and a spring, the period of the rotating rod is inversely proportional to $\sqrt{k}$. Our result indicates that the rod oscillates very rapidly, with a frequency $f = 1/T = 15.3\,\text{Hz}$, i.e., about 15 times in one second.