

47. We use Eq. 15-29 and the parallel-axis theorem  $I = I_{\text{cm}} + mh^2$  where  $h = d$ . For a solid disk of mass  $m$ , the rotational inertia about its center of mass is  $I_{\text{cm}} = mR^2/2$ . Therefore,

$$T = 2\pi \sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi \sqrt{\frac{R^2 + 2d^2}{2gd}} = 2\pi \sqrt{\frac{(2.35 \text{ cm})^2 + 2(1.75 \text{ cm})^2}{2(980 \text{ cm/s}^2)(1.75 \text{ cm})}} = 0.366 \text{ s.}$$

51. This is similar to the situation treated in Sample Problem 15.5 — “Physical pendulum, period and length,” except that  $O$  is no longer at the end of the stick. Referring to the center of mass as  $C$  (assumed to be the geometric center of the stick), we see that the distance between  $O$  and  $C$  is  $h = x$ . The parallel axis theorem (see Eq. 15-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right).$$

Equation 15-29 gives

$$T = 2\pi \sqrt{\frac{I}{mgh}} = 2\pi \sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi \sqrt{\frac{(L^2 + 12x^2)}{12gx}}.$$

(a) Minimizing  $T$  by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with  $12gT^2/2\pi$  instead of  $T$  (it should be clear that  $12gT^2/2\pi$  is a minimum whenever  $T$  is a minimum). The result is

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields  $x = L/\sqrt{12} = (1.85 \text{ m})/\sqrt{12} = 0.53 \text{ m}$  as the value of  $x$  that should produce the smallest possible value of  $T$ .

(b) With  $L = 1.85 \text{ m}$  and  $x = 0.53 \text{ m}$ , we obtain  $T = 2.1 \text{ s}$  from the expression derived in part (a).

92. The period formula, Eq. 15-29, requires knowing the distance  $h$  from the axis of rotation and the center of mass of the system. We also need the rotational inertia  $I$  about the axis of rotation. From the figure, we see  $h = L + R$  where  $R = 0.15 \text{ m}$ . Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M(L + R)^2,$$

where  $M = 1.0 \text{ kg}$ . Thus, Eq. 15-29, with  $T = 2.0 \text{ s}$ , leads to

$$2.0 = 2\pi \sqrt{\frac{\frac{1}{2}MR^2 + M(L+R)^2}{Mg(L+R)}}$$

which leads to  $L = 0.8315 \text{ m}$ .