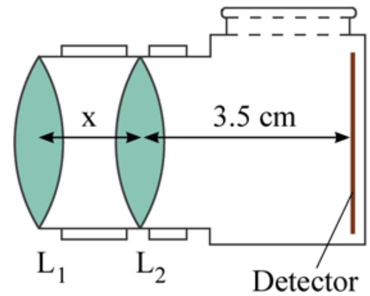
**Phyx 135-3 (1 pm) Midterm I** **Name:** ______________________________

**Winter 2017** **Discussion Section:** __________________

**1)** You have a digital camera in which one of the lenses, $L_2$, is at a fixed distance of 3.5 cm from the CCD detector. With only this lens in the camera, an object that is 5.25 cm to the left of $L_2$ can be perfectly focused on the detector. However, to photograph objects at other distances, a second lens $L_1$ (which is identical to lens $L_2$ in every way) must be placed in front of $L_2$. You notice that someone has left this distance set at $x = 1$ cm.




**1a) (3 points)** What is the focal length of $L_1$ and $L_2$?

**Solution**
We use $^1/_O + {}^1/_I = {}^1/_f$ to find the focal length of $L_2$ (and $L_1$): $^1/_{5.25} + {}^1/_{3.5} = {}^1/_f$, or $f = 2.1$ cm.

**1b) (4 points)** Where must $L_1$ cast an image (relative to $L_1$) in order to create a clear final image on the detector?

**Solution**
The "trick" here is to recognize that 5.25 cm to the left of $L_2$ is the object distance for $L_2$ when the object is physical. Therefore, it must *still* be the object distance (if I want to focus at 3.5 cm) even if the "object" is now an image created by $L_1$. The object for $L_2$ is also the image from $L_1$, so the $L_1$ image must be 5.25 cm – 1 cm = 4.25 cm to the left of $L_1$.

**1c) (3 points)** How far from $L_1$ was the object that the previous user was trying to photograph?

**Solution**
The image for $L_1$ is 4.25 cm to the left (negative image distance), so the object distance from $L_1$ is $^1/_O - {}^1/_{4.25} = {}^1/_{2.1}$, or $O = 1.4$ cm.

**2)** You have a tightly-stretched string which is simultaneously carrying two sin waves. The first wave has an amplitude of $y_1 = (0.1) \sin(1.2566\ x - 942.48\ t)$, and the second wave has an amplitude of $y_2 = (0.08) \sin(1.1729\ x - \omega t)$. (All units are in meters and seconds.)

**2a) (4 points)** What is the frequency of the second wave?

**Solution**

We realize that the two waves must be travelling at the same speed, which will be given by $v = \omega/k$ for either wave. For the first wave, $v = 942.48 / 1.2566 = 750$ m/s. Thus f for the second wave will be given by: $\omega = 2\pi f = vk$, or $f = (750)(1.1729)/2\pi =$ 140 Hz.

**2b) (6 points)** What is the amplitude of the string at x = 2.4 m if the first wave has completed exactly one cycle?

**Solution**

The frequency of the first wave is $f = \omega/2\pi = 942.48 / 2\pi = 150$ Hz, so its period is $^{1}/_{150} = 0.006667$ s. Thus we have x = 2.4 m and t = 0.006667 for both equations. Computing their amplitudes gives us:

$y_1 = (10\text{ cm}) \sin[(1.2566)(2.4) - (942.48)(0.006667)] = 1.25$ cm

$y_2 = (8\text{ cm}) \sin[(1.1729)(2.4) - (140)(2\pi)(0.006667)] = -0.74$ cm

The combined amplitude is 1.25 – 0.74 = 0.51 cm.

**3)** Laser light with $\lambda$ = 532 nm is sent down a long vacuum tube in a laboratory. The time of transit of the light from end-to-end in the tube is measured to be 17 ns. Then a glass cube that is 84 cm on a side is placed into the vacuum tube, and the time of transit of the light increases to 19.2 ns. Assume that the speed of light in a vacuum is $c = 3 \times 10^8$ m/s.

**3a) (6 points)** What is the index of refraction of the glass?

**Solution**

The transit time was increased by 19.2 – 17 = 2.2 ns. This represents the *extra* time it took the light to cross the 84 cm of the glass cube. The time for light to move 84 cm in a vacuum is exactly $t = (0.84 \text{ m})/(3 \times 10^8 \text{ m/s}) = 2.8 \times 10^{-9} \text{ s} = 2.8 \text{ ns}$, so the total time needed to cross the glass was 2.8 + 2.2 = 5 ns. Thus, the speed of light in the glass is only 2.8 / 5 = 56% of that in a vacuum, which means $n = 1 / 0.56 = 1.786$

**3b) (4 points)** About how many wave crests (of the electromagnetic field emitted by the laser) are there inside the glass cube at any given time?

**Solution**

The wavelength of the light inside the cube is 532 nm / 1.786 = 298 nm. Give or take, the number of wave crests inside the cube will be $84 \text{ cm} / 298 \text{ nm} = (0.84 \text{ m})/(2.98 \times 10^{-7} \text{ m}) = 2.82 \times 10^6$.

**4) (3 points)** Laser light with $\lambda$ = 660 nm is shining through two slits which are designed such that the distance between them can be adjusted. What distance should be chosen such that the m = 4 diffraction spot from the laser can just barely *not* be seen?

**Solution**

From $d \sin\theta = m\lambda$, we know that the m = 4 spot will just disappear when $\theta = 90°$. We therefore have $d = (4)(660 \text{ nm}) = 2.64 \times 10^{-6}$ m.

**5)** You have an organ pipe which can make sounds at f = 1372 Hz and at f = 1764 Hz. It cannot make any sound in between these frequencies.

**5a) (7 points)** What is the fundamental frequency of this pipe, and is it open at one end or open at both ends?

**Solution**

There are several ways to solve this. One is to realize that the frequencies of an open pipe are given by n = 1, 2, 3, etc, so successive harmonics must have the ratio (n + 1)/n. By contrast, the frequencies of a half-open pipe are given by n = 1, 3, 5, etc, so their ratio is (n + 2)/n. The ratio here is 1764/1372 = 1.286, so you could pop some ratios into your calculator and go until you find the right one:

| First Few Open Pipe Ratios | First Few Half-Open Pipe Ratios |
|---|---|
| 2/1 = 2.000 | 3/1 = 3.000 |
| 3/2 = 1.500 | 5/3 = 1.667 |
| 4/3 = 1.333 | 7/5 = 1.400 |
| 5/4 = 1.250 | 9/7 = 1.286 |

Obviously, n = 7 and n = 9 for a half-open pipe are the correct harmonics. This then yields a fundamental of either 1372/7 or 1764/9 = 196 Hz.

Another way of attacking this problem is to note that 1764 Hz – 1372 Hz = 392 Hz. Then you might assume that your fundamental frequency is 392 Hz. However, upon double-checking, you notice that 1372 / 392 = 3.5, and 1764 / 392 = 4.5, which are impossible, so you next guess that the real fundamental is 392 / 2 = 196 Hz. Then you have 1372 / 196 = 7 and 1764 / 196 = 9, which are whole numbers and therefore possible. The fact that the successive harmonics are two odd numbers tells you that you have a half-open pipe.

**5b) (3 points)** What is the length of the pipe?

**Solution**

f = nv/4L for a half-open pipe. We have L = nv/4f = (7)(345)/(4)(1372) = 0.44 meter.

**6) (7 points)** A swimming duck is paddling in a pool of water. It is moving its feet every 1.6 sec, making surface waves with this period. The speed of the surface waves is 0.32 m/s. If the duck is swimming at constant speed, and the crests of the surface waves ahead of the duck are spaced 0.12 meters apart, how fast is the duck moving?

**Solution**

The frequency of the waves coming from the duck is $f = 1/T = 1/1.6\text{ s} = 0.625\text{ Hz}$. The frequency of the waves ahead of the duck is $f = v/\lambda = (0.32\text{ m/s})/(0.12\text{ m}) = 2.667\text{ Hz}$. From the Doppler formula, we have $f = f_0 (v + 0) / (v - v_D)$, where the speed of the "listener" is zero (you are just watching the water waves) and the speed of the duck has a minus sign because we are looking at waves in front of it. Putting in numbers: $2.667 = (0.625)(0.32)/(0.32 - v_D)$, or $(0.32 - v_D) = (0.625)(0.32)/(2.667) = 0.075$, or $v_D = 0.245\text{ m/s}$.