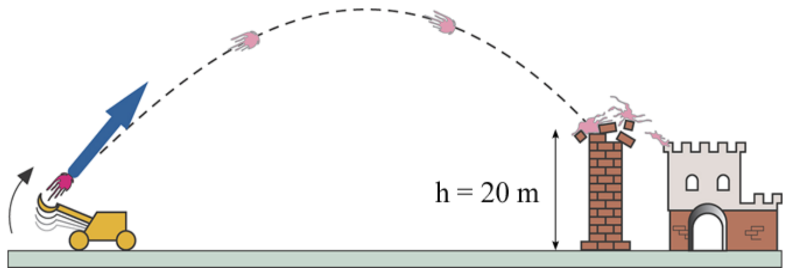
Phyx 135-1 Midterm I, Spring 2017 **Name** ________________________________

1) (10 points) You are using a catapult to throw flaming balls at a castle wall. If the catapult always launches the balls at a 55° angle and at a speed of 60 m/s, how far away from the castle must you be to land a flaming ball exactly on top of a wall that is 20 m high? (See illustration.)



Solution

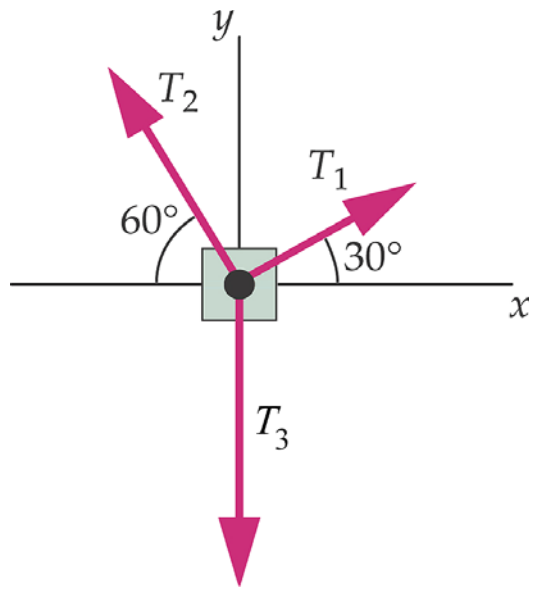
The most direct way to solve this problem is to start by writing down a general equation for the motion of the ball in the y-direction. We have: $y = y_o + v_o t + ½ at^2$. We can set $y_o = 0$, which then means that y = 20 m, $v_o = (60) \sin(55°) = 49.15$ m/s, and $a = g = –9.8$ m/s^2. This then gives us a quadratic equation of: $–4.9 t^2 + 49.15 t – 20 = 0$. Solving the equation yields:

$$–9.8 t = –49.15 \pm [49.15^2 – 4(–4.9)(–20)]^{1/2} = –49.15 \pm 44.99 = –94.14$$

We have chosen the negative half of the ± symbol because choosing the plus half gives you a very short time. This corresponds to when the flaming ball hits 20 m in height *on its way up*, but we want the time when the flaming ball is on its way down, so we select the longer time.

Thus we have t = 94.14 / 9.8 = 9.61 s. In this time the flaming ball will travel $d = v_x t$ = (60) cos(55°) (9.61) = 331 m.

2) (10 points) The illustration at right shows a top view of a table that is being manhandled by three rather inept movers that we shall call T_1, T_2, and T_3. The movers are pulling on the table with forces of $T_1 = 500$ N, $T_2 = 700$ N, and $T_3 = 800$ N. If the table has a mass of 50 kg, and is resting on a floor with a coefficient of kinetic friction $\mu_k = 0.15$, how fast is the table accelerating?



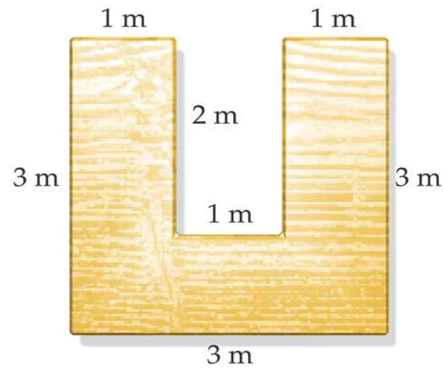
Solution

We first need to find the total force being applied to the table by the movers. In the x-direction, this will be:
$T_1 \cos(30°) – T_2 \cos(60°) = (500)(0.866) – (700)(0.5) =$ 83 N. In the y-direction, the applied force will be:
$T_1 \sin(30°) + T_2 \sin(60°) – T_3 =$
$(500)(0.5) + (700)(0.866) – 800 = 56.2$ N.

The total force from the movers is thus $(83^2 + 56.2^2)^{1/2} = 100.2$ N

The frictional force will exactly oppose this. We have $F = \mu_k N = \mu_k mg = (0.15)(50)(9.8) = 73.5$ N. So, the net force acting on the safe is 100.2 – 73.5 = 26.7 N, and therefore the acceleration of the table is 26.7 / 50 = 0.534 m/s.

3) (10 points) Where is the center of mass (both the x- and y-coordinates) of the object at right? You may assume that the density of the object is uniform in two dimensions. Please use the lower left-hand corner of the object as your coordinate zero.



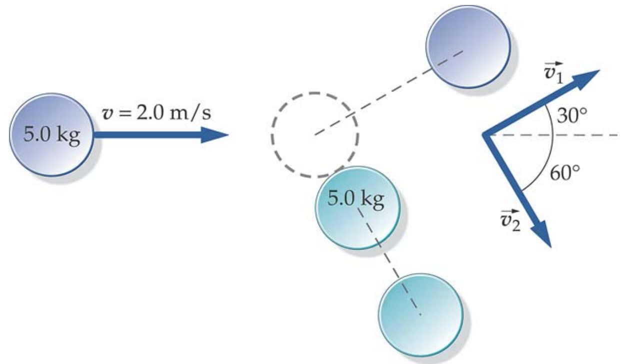
Solution

From symmetry, we see that the x-coordinate of the CM will be at x = 1.5 m.

For the y-coordinate, there are several ways to go about calculating y_{CM}. I will divide the "U" shape such that it has a bottom rectangle of (3 m X 1 m), and two top rectangles that are (1 m X 2 m). In other words, I am cutting straight across the bottom of the gap in the "U". This means that the bottom rectangle will have a mass of 3, and (by symmetry) have its y_{CM} at y = ½. Then, the two top rectangles will have masses of 2 each, and a y_{CM} at y = 2.

So, y_{CM} = [(0.5)(3) + (2)(2) + (2)(2)] / (3 + 2 + 2) = 9.5 / 7 = 1.36 m.

4) The illustration at right shows a top view of both the "before" and "after" configurations of two 5-kg pucks sliding on a frictionless surface. Before, the left puck is moving at 2 m/s directly along the x-axis, and the right puck is motionless. Afterwards, the left puck is moving at 30° upwards and the right puck is moving at 60° downwards.



4a) (7 points) What are the speeds v_1 and v_2 of the pucks after the collision?

Solution

From conservation of momentum, we know that the x-momentum of the "after" configuration must add up to mv = (5 kg)(2 m/s) = 10 kg m/s. That is, $10 = 5v_1 \cos(30°) + 5v_2 \cos(60°)$.

The y-momentum before the collision is zero, so it must also be the case that the "after" configuration has $mv_1 \sin(30°) = mv_2 \sin(60°)$. This yields $v_1(0.5) = v_2(0.866)$, or $v_1 = 1.732\ v_2$.

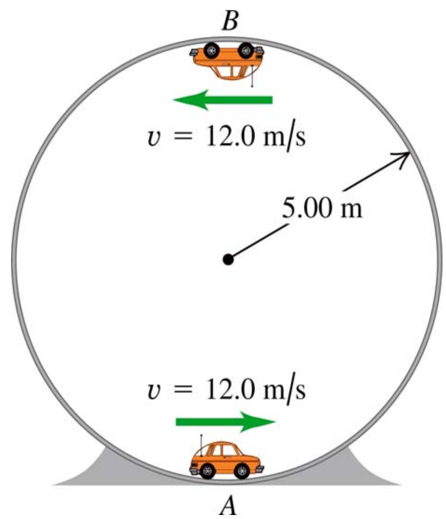
Substitution back into the first equation gets us: $2 = (1.732)v_2(0.866) + v_2(0.5)$, or v_2 = 1 m/s.
Then, v_1 = 1.732 m/s.

4b) (3 points) Is this collision elastic or not? (Guesses are not allowed; you must prove your answer with a calculation.)

Solution

The "before" kinetic energy is $E = ½\ mv^2 = ½\ (5)(2^2) = 10$ J. The "after" kinetic energy is
$E = ½(5)(1.732^2) + ½(5)(1^2) = 10$ J, so yes, the collision is elastic.

5a) (5 points) A little car is zipping around a vertical loop which has a radius of 5.00 m. The car is moving at 12 m/s. On the seat of the car there is a scale with a fish on it. If the scale reads 35 N when the car is at point A in the illustration, what is the mass of the fish?



Solution
The acceleration acting on the fish at point A will be equal to gravity plus the so-called centrifugal force. That is,
$a = g + v^2/r = 9.8 + 12^2 / 5 = 38.6 \text{ m/s}^2$. We thus have
$F = 35 \text{ N} = m(38.6)$, or m = 0.907 kg.

5b) (5 points) What does that same scale show when the little car is at point B?

Solution
At point B the centrifugal force and gravity will be working against each other, so the acceleration will be $v^2/r - g = 12^2 / 5 - 9.8 = 19 \text{ m/s}^2$. This means the scale will read $F = ma = (0.907)(19)$ = 17.2 N.